CONSISTENT FUZZY CONCEPT HIERARCHIES FOR ATTRIBUTE GENERALIZATION

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Abstract

In this paper we focus on attribute-oriented induction in data mining with fuzzy generalization hierarchies. We analyze in detail the process of data generalization through concept hierarchies, applying investigated properties for the fuzzy generalization model. We introduce a complete and a consistent model for fuzzy generalization, allowing the extraction of valid conclusions and a consistent attribute induction process.

Key Words: Knowledge Discovery in Databases, Attribute-oriented induction, Fuzzy Concept Hierarchies, Data Generalization

1. Introduction

Data generalization is a process of grouping of data, enabling transformation of similar data collections, expressed originally in a database at the low (primitive) level, into more abstract conceptual representations. This process is a fundamental element of attribute-oriented induction, a descriptive database mining technique, which compresses the original set of data into a generalized relation, providing concise and summarative information about the massive set of task-relevant data.

Generalization of database records is performed on an attribute-by-attribute basis, applying a separate concept hierarchy for the each of the generalized attributes included in the relation of task-relevant data. Each concept hierarchy represents background knowledge about the domain allowing gradual, similarity-based, aggregation of attribute values stored in the original tuples. This hierarchy is built usually in the bottom-up manner progressively increasing the abstraction of the generalization concepts at each new level. Using Wah's [1] terminology this process can be characterized as a knowledge-intensive method (since the concept hierarchies are based on the external knowledge provided during their creation), but with a strong data-oriented approach (since these hierarchies are built from bottom to top to embrace only the attribute values, occurring in the database). Creation of new concept levels in generalization hierarchies is accompanied by an increase of the concept abstraction and the decrease of their cardinality (each higher level includes less data descriptors, but they have more general meaning).

Depending on the approach and the intention of data analysts, generalization of collected data is considered to be both a final step of data mining (e.g. summary-tables are presented to users, allowing them to interpret overall information [2, 3, 4]), as well as an introduction to further knowledge extraction (e.g. extraction of abstract association rules directly from the generalized data [5, 6]).

Attribute generalization should not be mistaken for simple record summarization. Summaries of data usually have a much more simplified character – all data is typically presented with the concepts at the same level of abstraction (lacking hierarchical concept structure characteristic for attribute generalization) or no abstraction of values is introduced (simply reporting). Moreover records, which do not occur in large quantities, are usually omitted to simplify the final report. Gradual generalization through concept hierarchies allows in contrast detailed tracking of all records, and can lead to the discovery of interesting patterns among data at the lowest possible abstraction level of their occurrence, decreasing at the same time the risk of omitting them due to over-generalization.

Despite the dynamic progress in the research conducted currently on data mining algorithms, the phase of data generalization remains a crucial activity. The choice of data to be analyzed as well as of the concepts for its generalization has a fundamental influence on retrieved results, regardless of applied knowledge acquisition techniques. With data generalization, executed at the initial stage of data mining, the process of knowledge extraction can be more effective at the abstraction level desired by a user.

In this paper we focus on fuzzy generalization hierarchies, as they seem to better capture human approaches to representing generalizations. Typically the more abstract the level of concepts to be used in data generalization, the less certain experts are at assigning particular lower-level concepts to them. This situation can be naturally modeled utilizing the concept of partial memberships from fuzzy set theory. There has been some research applying fuzzy concept hierarchies to the data summarization task [7, 8, 9, 10]; however these approaches lack certain properties that we believe are desirable for the attribute oriented data mining process.

In the next section we provide a short overview of research on data generalization, emphasizing work done with fuzzy set approaches. Then we present the concept of fuzzy generalization hierarchies and investigate desired properties of generalization. Here we also introduce a complete and consistent model of a fuzzy generalization hierarchy and study its properties as related to attribute generalization.

2. Related work

The idea of applying concept hierarchies for attribute-oriented induction in data mining was popularized by Han and his co-researchers [2, 3, 4, 5] and extended further by Hilderman, Hamilton, Cecrone and their co-workers [11, 12, 13]. Yager [14, 15], followed by Kacprzyk [16] and Dubois and Prade [17], investigated application of fuzzy sets to the area of dataset summarization through linguistic concepts. Hierarchies for fuzzy summarization of database records were approached directly in late nineties by four groups of independent researchers. Lee and Kim [7] used ISA hierarchies, from area of data modeling, to generalize database records to more abstract concepts. Lee [10] applied fuzzy generalization hierarchies to mine generalized fuzzy quantitative association rules. Cubero, Medina, Pons and Vila [9] presented fuzzy gradual rules for data summarization and Raschia, Ughetto and Mouaddib [8] implemented SaintEtiq system for data summarization through extended concept hierarchies.

The fuzzy summarization hierarchies, introduced so far, despite their ability to better reflect real-life dependencies, have difficulty with exact induction from data. The problem, as we believe, lays in the lack of model completeness, where a single tuple is guaranteed to remain with a "weight" of one record at each of the fuzzy generalization levels. In this paper we study this problem and propose a solution assuring preservation of model completeness at each of the levels in the fuzzy generalization hierarchy.

3. Consistent fuzzy generalization hierarchy

3.1. Formal definition of fuzzy concept hierarchy

We can define a fuzzy concept hierarchy *FCH* as an ordered pair (C, L), where C is a set of concepts utilized to generalize a particular domain and L is a set of links between these concepts, reflecting ideas applied for the generalization process. Each concept c has its unique name (label) and abstraction level, placing it on a specific height of the generalization hierarchy. A single concept in the generalization hierarchy can be described as a pair (v, j) representing a node and a hierarchy (abstraction) level in the generalization tree. To simplify notation we denote this as v^j and to refer directly to the specific concept at the given level of the generalization hierarchy we use v_i^j, where i symbolizes the index of the concept v at the j^{th} abstraction level.

Normally, the depth level of a rooted tree is calculated downward starting from the root, where root is denoted as level 0; in our approach we use an upward numeration. This seems to be more natural for generalization hierarchies since it reflects directly the conceptual abstraction level (original attribute values, presented at the bottom of such hierarchy, have level 0 – which corresponds exactly to the lack of data abstraction). Usually a bottom-up generalization process is run continuously until the cardinality of generalized records exceeds the given threshold value (e.g. 5% of the analyzed population), and so we may not know the total height of the generalization hierarchy until the process is completed.

A link l in the fuzzy concept hierarchy is a directed arc (edge) between two nodes with a certain weight assigned to it. Such a structure can be described as a triple (s, t, μ_{st}), where s and t are endpoints of l, and μ_{st} represents the share with which the single lower-level concept s belongs to the more general descriptor t. The concept s is called a source of l and always placed on the lower level of abstraction in the concept hierarchy and will be termed a *direct specializer of t*. The second endpoint t is called commonly a target of l; we will refer to it as to a *direct abstract of s*. Weight μ_{st} represents grade of membership of concept s to its direct abstract t. In other words μ_{st} reflects the strength of conviction that the lower-level concept s should be qualified as concept t when the data generalization process moves to the next abstraction level.

Link l, which connects s and t with strength μ_{st} symbolizes the *generalization relation*, denoted here as $s \succ t \mid \mu_{st}$. Generalization based on the concept hierarchies will be performed directly only between adjacent abstraction levels. So when s symbolizes a certain concept at the abstraction level j (s^j), then t, being a direct abstract of s, has to be placed exactly one level above (t^{j+1}). An abstraction level, which is in the generalization hierarchy placed right above the certain one, is to be called the *direct abstraction level*. The one below will be called a *direct specialization level*.

The main difference between a crisp generalization hierarchy and a fuzzy one, applicable for the attribute induction in data mining, is the type of generalization relation between the concept and its direct abstract. In crisp hierarchies, each concept s is fully assigned to only one direct abstract, however a single abstract can have many direct specializers, a *many-to-one* relation. In fuzzy concept hierarchies a generalization relation can have a *many-to-many* character, where a single concept s can have more than one direct abstract t and s can belong to the higher-level concept t to a certain extent, expressed by μ_{st}, $\mu_{st} \in [0,1]$.

Applying a fuzzy approach to data generalization, we define each abstract concept (a concept at the hierarchy level higher than 0) as a fuzzy set over a domain space defined by the original attribute values. Depending on the current abstraction level in the concept hierarchy a generalized space (domain) may be represented by the original values stored in the database records (Figure 1) or by the concepts already applied for the data generalization. In fact a construction of a new level in the generalization hierarchy should be understood as a gradual defining of new and more extensive fuzzy sets (more abstract concepts) over the domain space represented by the concepts (possibly also fuzzy sets) coming from the hierarchy one-level below the one which is currently being built.

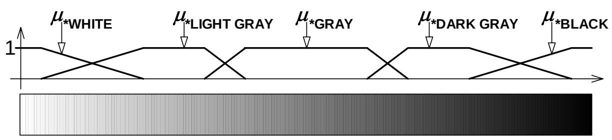

Figure 1. Construction of the fuzzy sets in the space of domain ACHROMATIC COLOR.

To build the next level of abstraction for the fuzzy generalization hierarchy we have to introduce new concepts, which have more general meaning than those used at the first level. So we chose three linguistic descriptors: WHITISH, GRAYISH and BLACKISH and extended them to form the fuzzy sets by designing membership functions $\mu_{*WHITISH}$, $\mu_{*GRAYISH}$, $\mu_{*BLACKISH}$, as presented in Figure 2 (the space of original attribute values remains to be entirely covered by these three new-defined fuzzy sets). For each of the lower level concepts the membership functions had different values. For WHITE we have: $\mu_{WHITE\text{-}WHITISH}=1$, $\mu_{WHITE\text{-}GRAYISH}=0$, $\mu_{WHITE\text{-}BLACKISH}=0$; for LIGHT GRAY: $\mu_{L.GRAY\text{-}WHITISH}=0.3$, $\mu_{L.GRAY\text{-}GRAYISH}=0.7$, $\mu_{L.GRAY\text{-}BLACKISH}=0$; for GRAY: $\mu_{GRAY\text{-}WHITISH}=0$, $\mu_{GRAY\text{-}GRAYISH}=1$, $\mu_{GRAY\text{-}BLACKISH}=0$; for DARK GRAY: $\mu_{D.GRAY\text{-}WHITISH}=0$, $\mu_{D.GRAY\text{-}GRAYISH}=0.7$, $\mu_{D.GRAY\text{-}BLACKISH}=0.3$; for BLACK: $\mu_{BLACK\text{-}WHITISH}=0$, $\mu_{BLACK\text{-}GRAYISH}=0$, $\mu_{BLACK\text{-}BLACKISH}=1$.

Such approach can be continued to achieve more advanced and higher structures applicable for the generalization of data. In Figure 3 we present a possible 5-level generalization hierarchy for domain of ACHROMATIC COLORS.

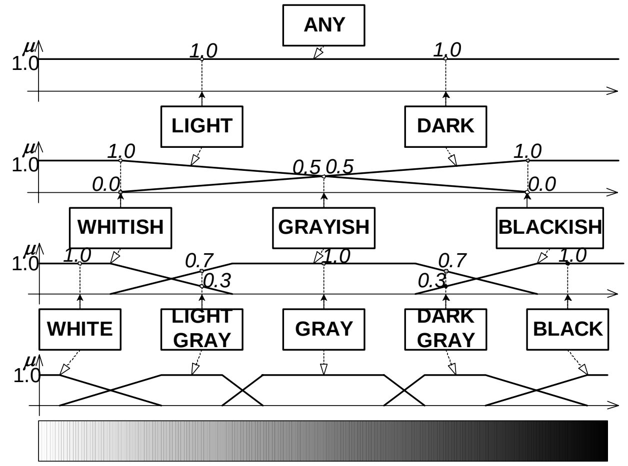

Figure 2. Hierarchical model of fuzzy sets in the domain ACHROMATIC COLOR.

The generalization hierarchy tree presented in Figure 3 was constructed on the basis of the hierarchy of fuzzy sets applied for generalization of the domain, as presented in Figure 2. A gradually extended meaning of abstract concepts allows all the space of domain to be covered at each of the hierarchy levels. So the fuzzy sets that are used as a representation of generalization concepts must have a broading character at each of the more abstract levels. At the top of the hierarchy, the linguistic concept ANY is represented by one set which covers entire space of the generalized domain.

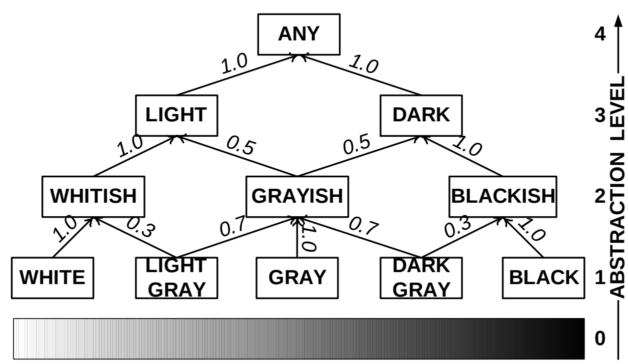

Figure 3. Fuzzy generalization hierarchy for continuous attribute values of domain ACHROMATIC COLOR.

3.2. Desired properties of data-mining generalization

Attribute-oriented generalization in data mining performs grouping of tuples based on the similarity of the attribute values. The tuples, which fall into the same generalization category, are represented by a generalized record together with a count of the records it represents. There are some basic properties of attribute-oriented generalization, which must to be maintained to ensure valid results.

(1) Generalization applied for attribute induction purposes should be *irreflexive*. No single concept in the generalization hierarchy can become a direct or indirect abstract (generalized descriptor) of itself. It can be formally described as follows:

$$\forall s \in C^{i}, \exists t \in C^{i+1} \text{ s.t. } s \succ t | \mu_{st} \Rightarrow t \neq s$$

where $\mu_{st} \neq 0, i < k-1$; k is the height of the generalization hierarchy, and C^{i+1} is a direct abstraction level of C^{i} in the generalization hierarchy.

The rationale here is quite obvious – there is simply no sense in generalizing the attribute value to an identical one. The grouping of tuples with an identical attribute value can be achieved without the necessity of employing the generalization hierarchy. Reflexive generalization could introduce confusion, since when analyzing results of the data generalization we would not be able to easily clarify which level of abstraction the duplicated concept represents. Moreover, the duplication of the same concepts at the different levels of hierarchy is in conflict with the idea that abstraction increases with the rise of generalization hierarchy level.

The only case where this situation might occur would be aggregation of tuples with the values named differently but having exactly the same meaning – synonyms. If we would like to generalize values from the domain MEANS OF TRANSPORTATION, words such as *car, automobile, auto* would be instantly unified by a human, however we would need external knowledge to have them generalized by the computer. The unification automatically performed by a human would probably assign the conceptual meaning of a car (as it is currently the most common word) to all three concepts, allowing the occurrence of reflexive relation: $car \succ car \mid 1.0$. At this point it is important to emphasize the difference between unification and generalization. Despite the fact that both of these relations group values according to their similarity, there is no element of abstracting from the values involved in the unification process. We do not believe unification should be considered as a part of generalization, but rather should occur as a step of the data preprocessing.

To have the fuzzy generalization model maintain this property we must assure that the concept labels are unique and that the abstraction of concepts used for data generalization gradually increases along the hierarchy levels.

(2) No abstract in the generalization hierarchy can have only one direct specializer. In other words, *a minimum of two concepts must be generalized* if we want to move to the higher level of abstraction in the generalization hierarchy. Using notation as above such property may be formally denoted as:

$$\forall s \in C^{i} \exists t \in C^{i+1} \text{ s.t. } s \succ t | \mu_{st} \Rightarrow$$
$$\Rightarrow \exists u \in C^{i} \text{ s.t. } s \neq u \wedge u \succ t | \mu_{ut}$$

where $\mu_{st}, \mu_{ut} \neq 0; i < k-1$

This property prevents transformation of a single set of identical attribute values into a more abstract form, which could cause problems. Assume we generalize a single value *car* from the domain MEANS OF TRANSPORTATION to the concept *motor vehicle*, and there are no other attribute values in the data which could be generalized to this concept at the moment. Analyzing in parallel the domain NUMBER OF WHEELS, we could easily end up with conclusion that *All motor vehicles have at least four wheels*, which is obviously erroneous. However if we use values from the lower level of the generalization hierarchy we conclude with the rule *All cars have at least four wheels*, which is correct and has exactly the same confidence and support as the first one. Generalization of only one concept in the hierarchy to a higher-level concept pointlessly distances us from the original information stored in the database and may often lead to erroneous impression about data, when the abstract of the concept has a wider meaning than intended.

(3) Generalization has *asymmetric* character:

$$\forall s \in C^{i} \exists t \in C^{i+1} \text{ s.t. } s \succ t | \mu_{st} \Rightarrow \neg(t \succ s | \mu_{ts})$$

where $\mu_{st} \neq 0; \mu_{ts} \neq 0; i < k-1$

This property reflects the unidirectional character of generalization, because the level of abstraction constantly increases with each generalization step. This is the property, distinguishing generalization from unification, which has a symmetric character.

A lack of links going downwards and the uniqueness of the concept labels in the fuzzy concept hierarchies guarantee the asymmetry of fuzzy generalization.

(4) Generalization should be *transitive*. If a specific concept is a generalization of another concept it *must be* also as a generalization of the specializer of that concept. Formally:

$$\forall s \in C^{i} \exists t \in C^{i+1} \exists u \in C^{i+2} \text{ s.t. } s \succ t | \mu_{st} \wedge t \succ u | \mu_{tu} \Rightarrow$$
$$\Rightarrow s \succ u | \mu_{su}$$

where $\mu_{st}, \mu_{tu}, \mu_{su} \neq 0$ and $i < k-2$

Transitivity allows efficiency of data generalization that has to be performed repeatedly on the same domain. Applying transitivity allows reduction of the generalization hierarchy height, by skipping over abstraction levels that did not provide a meaningful generalization. We can then compress the generalization hierarchy to the minimal but most efficient height, where at each abstraction level we have at least one generalized record including the number of the original tuples above the given generalization threshold.

Such hierarchy size reduction, based on the transitivity of the generalization relation is possible in a complete and consistent fuzzy model and will be discussed next.

3.3. Consistency and completeness of the fuzzy generalization model

To be able to retrieve exact information from data that has been generalized we have to keep track of all records throughout the whole generalization process, independently of the abstraction level. When calculating support or confidence of mined relationships among data we want to be certain that no record or group of records were accidentally or purposely omitted or duplicated during generalization. At each level of abstraction in the generalization hierarchy a single tuple needs to be counted exactly once. When non-fuzzy generalization is utilized for attribute induction this requirement can be automatically maintained by fulfilling two conditions at each of the hierarchy levels:

(1) The space of generalized domain described (covered) by the concepts at the each abstraction level has to be equal to the space represented by the domain values originally stored in the database. This condition preserves omitting tuples during generalization.

(2) All crisp concepts at each of the hierarchy levels are disjoint. This guarantees that no tuple is going to be processed more than once when moving from one abstraction level to another.

The requirement of single occurrence of every tuple at each of the abstraction levels can also be preserved in fuzzy generalization hierarchies. The first of the conditions is maintained during fuzzy generalization as in the case of crisp hierarchies – by adding new concepts for the currently built hierarchy level until all descriptors from the lower-level of generalization hierarchy are embraced. However, to guarantee the second requirement – no single tuple to be counted more than once at each of the hierarchy levels of the fuzzy generalization hierarchy, we have to assure completeness of the fuzzy generalization model.

An advantage of a fuzzy generalization hierarchy is the idea that abstract concepts used in the data generalization purposes do not have to be strictly disjoint. They are allowed to be fuzzy – certain concepts from the same abstraction level may now overlap to a certain extent. In Figure 1 we can observe that a tuple with a great amount of luminance stored in it is going to be partially generalized as a WHITE record and partially as a LIGHT GRAY one at the same time. Such overlapping seems to more accurately reflect a human way of thinking and fits well into generalization of many domains. However, to maintain the identical number of records at each level of the generalization hierarchy a fuzzy generalization model has to preserve its completeness.

The fuzzy generalization hierarchy is *complete and consistent* when for all adjacent hierarchy levels, *S* and *T* (where *T* is a direct abstraction level of *S*), the following relationship is satisfied:

$$\sum_{i=1}^{\|T\|} \mu_{st_i} = 1,\ \forall s \in S, \forall t_i \in T$$

where: *i* is the index of the generalization concepts at the abstraction level *T*.

In other words, the sum of weights assigned to the links leaving a single node in the fuzzy concept hierarchy needs to be always 1.0 to preserve completeness of the generalization model.

This property, when consistently maintained through all adjacent levels in the fuzzy generalization hierarchy remains valid when the transitivity of generalization is exploited to create a new fuzzy generalization model which would be shallower than the one initially established.

Each concept *s* placed at the level *j* of the fuzzy generalization hierarchy (s^j), which has *n* direct abstracts ($t_1^{j+1}, t_2^{j+1}, \ldots, t_n^{j+1}$) can be generalized to the concept at the abstraction level above the one it is adjacent to (u^{j+2}), using the following formula:

$$\forall s^j \exists t_i^{j+1}, u^{j+2} \text{ s.t. } s^j \succ t_i^{j+1}\big|\mu_{st_i} \wedge t_i^{j+1} \succ u^{j+2}\big|\mu_{t_iu} \Rightarrow$$
$$\Rightarrow s^j \succ u^{j+2}\big|\mu_{su}$$

where $\mu_{st_i}, \mu_{t_iu} \neq 0$ and $\mu_{su} = \sum_{i=1}^{n} \mu_{st_i} \cdot \mu_{t_iu}$

As we can see from the above a transitivity of the fuzzy generalization from the concept *s* to *u* can be maintained applying algebraic sum (s-norm operator) over algebraic product (t-norm operator) of the membership grades from all paths linking this two concepts in the fuzzy generalization hierarchy. These operators are generally more effective for attribute-oriented induction.

Speaking less formally – the weights of all paths leading from concept s^j to its indirect abstract u^{j+2} have to be multiplied and then the products of all itineraries between s^j and u^{j+2} have to be summed up. The result will be the weight representing the strength of a direct generalization idea (direct link), leading from the concept *s* to its abstract *u* at the level two stages higher in the generalization hierarchy.

An example of transitivity of the fuzzy generalization in the domain of ACHROMATIC COLOR is presented in Figure 4, where concepts at the level 1 of the fuzzy generalization hierarchy are transferred (generalized) directly to the level 3 of the previously presented hierarchy.

As we can see in Figure 3 the concept LIGHT GRAY at the level 1 of the generalization hierarchy is generalized partially to its direct abstracts WHITISH and GRAYISH ($\mu_{L.GRAY\text{-}WHITISH}=0.3$, $\mu_{L.GRAY\text{-}GRAYISH}=0.7$). The concept WHITISH is further generalized to its direct abstract LIGHT ($\mu_{WHITISH\text{-}LIGHT}=1$), and the GRAYISH is evenly generalized to both concepts at the third level of the hierarchy: LIGHT and DARK ($\mu_{GRAYISH\text{-}LIGHT}=0.5$ and $\mu_{GRAYISH\text{-}DARK}=0.5$). Applying transitivity of the fuzzy generalization relation through both of these generalization itineraries (paths) we can generalize LIGHT GRAY directly to the concepts at the third level of abstraction in the generalization hierarchy:

(1) from L.GRAY to LIGHT:

$$\mu_{L.GRAY\text{-}LIGHT}=\mu_{L.GRAY\text{-}WHITISH}*\mu_{WHITISH\text{-}LIGHT}+$$
$$+\mu_{L.GRAY\text{-}GRAYISH}*\mu_{GRAYISH\text{-}LIGHT}=0.3*1.0+0.7*0.5=0.65$$

(2) from L.GRAY to DARK:

$$\mu_{L.GRAY\text{-}DARK}=\mu_{L.GRAY\text{-}GRAYISH}*\mu_{GRAYISH\text{-}DARK}=0.7*0.5=0.35$$

We need to emphasize here that sum of weights at all links leaving the concept L.GRAY (denoted as $\mu_{L.GRAY-*}$) in the fuzzy hierarchy remains exactly 1. This preserves and guarantees the completeness of the generalization model after its height reduction:

$$\mu_{L.GRAY-*}=\mu_{L.GRAY-LIGHT}+\mu_{L.GRAY-DARK}=0.65+0.35=1.0$$

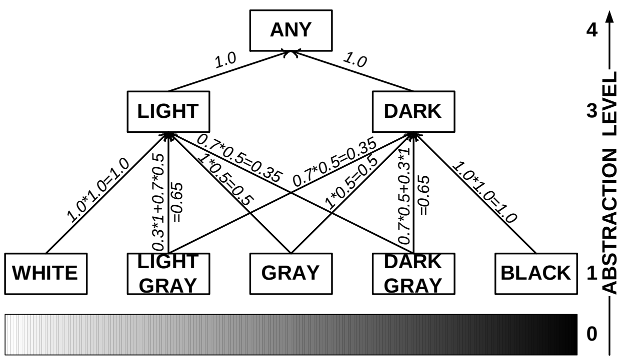

Figure 4. Transitivity of generalization.

4. Conclusion

We presented here a complete and consistent model of fuzzy generalization and studied its properties. Crisp generalization hierarchies fall into this model specification as its special case (where the only links between generalization concepts, which has weight equal 1, are allowed). Completeness of the fuzzy model presented in this paper guarantees the validity of the information retrieved when applied in data mining purposes. At the same time the model has advantages over comparable crisp hierarchies by allowing one to better reflect relations between lower-level concepts and their abstracts. Study of ontological relations between all concepts in the generalization hierarchy remains an interesting area for the further research.

References

[1] B.W. Wah, Generalization and Generalizability Measures, *IEEE Transactions On Knowledge And Data Engineering, 11*(1), 1999, 175-186.

[2] J. Han & M. Kamber, *Data Mining: Concepts and Techniques* (New York, NY: Morgan Kaufmann, 2000).

[3] J. Han & Y. Fu, Exploration of the Power of Attribute-Oriented Induction in Data Mining, in U.M. Fayyad, G. Piatetsky-Shapiro, P. Smyth & R. Uthurusamy (Eds.)**,** *Advances in Knowledge Discovery and Data Mining* (Menlo Park, CA: AAAI/MIT Press, 1996) 399-421.

[4] J. Han, Y. Cai, & N. Cercone, Knowledge discovery in databases: An attribute-oriented approach, *Proc. 18th Int. Conf. Very Large Data Bases*, Vancouver, Canada, 1992, 547-559.

[5] J. Han, Mining Knowledge at Multiple Concept Levels, *Proc. 4th Int'l Conf. on Information and Knowledge Management*, Baltimore, Maryland, 1995, 19-24.

[6] R. Srikant & R. Agrawal, Mining Generalized Association Rules, *Proc. 21st Int'l Conference on Very Large Databases*, Zurich, Switzerland, 1995, 407-419.

[7] D.H. Lee and M.H. Kim, Database summarization using fuzzy ISA hierarchies, *IEEE Transactions On Sysems, Man, and Cybernetics - part B, 27*(1), 1997, 68-78.

[8] G. Raschia, L. Ughetto, N. Mouaddib, Data summarization using extended concept hierarchies, *Proc. Joint 9th IFSA World Congress and 20th NAFIPS Int'l Conf.*, Vancouver, BC, Canada, 2001, 2289 -2294.

[9] J. C. Cubero, J.M. Medina, O. Pons, M.A. Vila, Data Summarization in Relational Databases Through Fuzzy Dependencies, *Information Sciences, 121*(3-4), 1999, 233-270.

[10] K.-M. Lee, Mining generalized fuzzy quantitative association rules with fuzzy generalization hierarchies, *Proc. Joint 9th IFSA World Congress and 20th NAFIPS Int'l Conf.*, Vancouver, Canada, 2001, 2977 -2982.

[11] H.J. Hamilton, R.J. Hilderman & N. Cercone, Attribute-oriented induction using domain generalization graphs, *Proc. 8th IEEE Int'l Conf. on Tools with Artificial Intelligence*, Toulouse, France, 1996, 246-253.

[12] C. L. Carter & H. J. Hamilton, Efficient Attribute-Oriented Generalization for Knowledge Discovery from Large Databases, *IEEE Transactions on Knowledge and Data Engineering, 10*(2), 1998, 193-208.

[13] R.J. Hilderman, H.J. Hamilton & N. Cercone, Data mining in large databases using domain generalization graphs, *Journal of Intelligent Information Systems, 13*(3), 1999, 195-234.

[14] R.R. Yager, On linguistic summaries of data, in W. Frawley & G. Piatetsky-Shapiro (Eds.), *Knowledge Discovery in Databases* (Menlo Park, CA: AAAI/MIT Press, 1991) 347-363.

[15] D. Rasmussen & R.R. Yager, A Fuzzy SQL Summary Language for Data Discovery, in D. Dubois, H. Prade and R.R. Yager (Eds.), *Fuzzy Information Engineering: A Guided Tour of Applications* (New York, NY: John Wiley and Sons Inc., 1997) 253-264.

[16] J. Kacprzyk, Fuzzy logic for linguistic summarization of databases, *Proc. 8th Int'l Conf. on Fuzzy Systems*, Seoul, Korea, 1999, 813-818.

[17] D. Dubois & H. Prade, Fuzzy sets in data summaries - outline of a new approach, *Proc. 8th Int'l Conf. on Information Processing and Management of Uncertainty in Knowledge-Based Systems*, Madrid, 2000, 1035-1040.