

CS 350 Theory of Computation

Assignment 1 (8 marks)

Question 1 (1 marks)

Prove that $1^2 + 3^2 + 5^2 + \dots + (2n - 1)^2 = \frac{1}{3}n(4n^2 - 1)$.

Question 2 (1 marks)

Given a planar graph $P = (V, E)$, we have Euler's formula: $|V| + |F| - |E| = 2$, where F is the set of faces of P . Let $|V| = n$. Prove that $|F|$ is at most $2n$.

Question 3 (1 marks)

Show that in any simple graph there is a path from any vertex of odd degree to some other vertex of odd degree.

Question 4 (1 marks)

Given a graph $G = (V, E)$, we define $w(G)$ as the number of connected components in G . A *cut edge* of G is an edge $e \in E$ such that $w(G - \{e\}) > w(G)$. (Definition: $G - \{e\} = (V, E - \{e\})$.)

Prove that $e \in E$ is a cut edge of G if and only if e is contained in no cycle of G .

Question 5 (2 marks)

A fully binary tree T is a tree such that all internal nodes have two children. Prove that a fully binary tree with n internal nodes in total has $2n + 1$ nodes.

Question 6 (2 marks)

Given an undirected graph $G = (V, E)$, the breadth-first-search starting at $v \in V$ ($bfs(v)$ for short) is to generate a shortest path tree starting at vertex $v \in V$. The diameter of G is the longest of all shortest paths $\delta(u, v), u, v \in V$.

When G is a tree, the following algorithm is proposed to compute the diameter of G .

1. Run $bfs(w), w \in V$ and compute the vertex $x \in V$ furthest from w .
2. Run $bfs(x)$ and compute the vertex $y \in V$ furthest from x .
3. Return $\delta(x, y)$ as the diameter of G .

Prove that this algorithm is correct; i.e., $\delta(x, y)$ is in fact the longest among all the shortest paths between $u, v \in V$.

Date Due: before the end of class on **Thursday, Feb 5, 2009**. No late assignment will be accepted.