Lemma. If a language is context free, then some PDA recognizes it.

IDEA. a) If $A$ is a CFL, then there is a corresponding CFG $G$.
   b) We need to convert $G \Rightarrow PDA$

Difficulties:
   c) When we use $G$ to derive some strings, the orders of apply rules are arbitrary. However, with a PDA, its stack can only be accessed from the top.

Solution

Try to always keep the top of a stack a variable (initially it is $S$), and this can be done by matching terminals on the stack with symbols in the input string.
$q_{\text{start}} \rightarrow S$

$S \rightarrow S \$ \quad \text{//}$S$ is placed on the stack$

$\varepsilon, A \rightarrow \omega$

If stack top is variable $A$, select one of the rules for $A$ and substitute $A$ by the right-hand side string.

$a, a \rightarrow \varepsilon$

If stack top is a terminal $a$, read the input and compare them. If they match, pop the stack.

$a, s \rightarrow x y z$

$r$