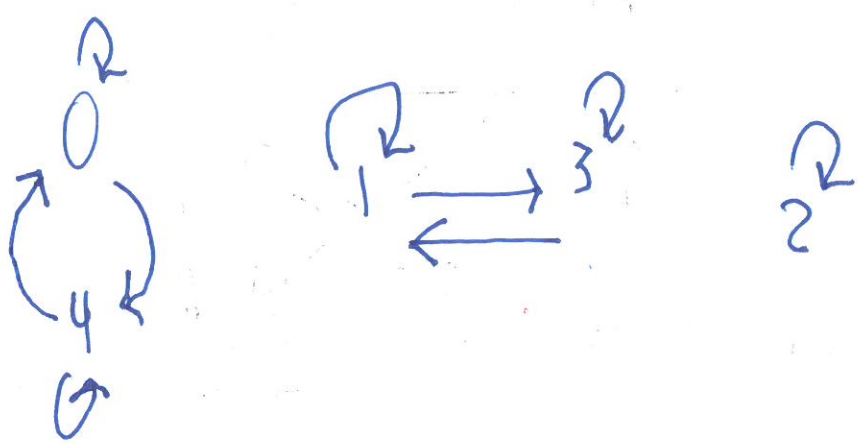
3.



$[0] = \{0, 4\} = [4]$

$[1] = \{1, 3\} = [3]$

$[2] = \{2\}$

13. $[aa**] = \{aaaa, aaab, aaba, aabb\}$

$[aaaa] = [aaab] = [aaba] = [aabb]$

$[ab**]$

$[ba**]$

$[bb**]$

21. reflexive

Consider a particular but arbitrarily chosen $x \in \mathbb{Z}$

is $x \, R \, x$ even?

is $7x - 5x$ even?

$7x - 5x = 2x$

since any integer multiplied by 2 is even
and x is an integer. $2x$ must be even

Symmetry

Consider particular but arbitrary chosen $x, y \in \mathbb{Z}$

~~...~~ such that $x \, R \, y$ ~~is even~~ produces an even # when $7x - 5y$

is $y \, R \, x$ ~~even~~?

is $7y - 5x$ ~~even~~?

$$7y - 5x = z \quad \text{is } z \text{ even?}$$

given $$7x - 5y = 2w \quad \text{where } w \in \mathbb{Z}$$

$$2x + 2y = z + 2w$$

$$2x + 2y - 2w = z$$

$$2(x + y - w) = z$$

z must be even because $2 \cdot (x + y - w)$ where $x, y, w \in \mathbb{Z}$ is even

[both #s are even]

[both #s are odd]

42b. Suppose that (a,b) and (c,d) are particular but arbitrarily chosen elements of A

Furthermore, suppose that $(a,b) \, R \, (c,d)$

We need to show $(c,d) \, R \, (a,b)$

---

by definition of R $\quad ad = bc$

by definition of $=$ $\quad bc = ad$

Commutativ ~~[illegible]~~ $\times$ $\quad \boxed{c}b = \boxed{d}a$

by definition of R $\quad (c,d) \, R \, (a,b)$