5. Prove that for all sets $A$ and $B$, $B - A = B \cap A^c$

(1) show that $B - A \subseteq \boxed{B \cap A^c}$

suppose that $x$ is a particular but arbitrarily chosen element of $B - A$, i.e. $x \in B - A$

$\therefore$ $x \in B$ and $x \notin A$ — by defn of set difference

$\therefore$ $x \in B$ and $x \in A^c$ — by defn of complement

$\therefore$ $x \in B \cap A^c$ — by definition of intersection

$\therefore$ $B - A \subseteq B \cap A^c$

(2) show that $B \cap A^c \subseteq B - A$

$\vdots$

$\therefore$ $B \cap A^c \subseteq B - A$

$\therefore$ $B - A = B \cap A^c$ — defn of set equality

15. Prove that for every set $A$, $A \cup \emptyset = A$

(1) show $A \cup \emptyset \subseteq A$

let $x$ be a particular but arbitrary element of $A \cup \emptyset$,
i.e. $x \in A \cup \emptyset$

$\therefore$ $x \in A$ or $x \in \emptyset$ by defn of union

however, $x \notin \emptyset$ by defn of $\emptyset$

$\therefore$ $x \in A$

$\therefore$ $A \cup \emptyset \subseteq A$

(2) show $A \subseteq A \cup \emptyset$

⋮

$\blacksquare$

Since $A \cup \emptyset \subseteq A$ and $A \subseteq A \cup \emptyset$, $A \cup \emptyset = A$
by defn of set equality

30. Prove that for every subset $A$ of universal set $U$,

$A \cap A^c = \emptyset$

Suppose $A \cap A^c \neq \emptyset$

then we can find a particular, but arbitrary element $x$ such that $x \in A \cap A^c$

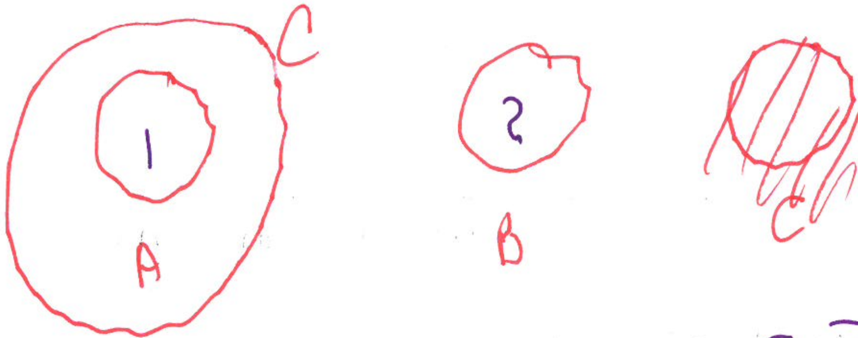
$\therefore$ $x \in A$ and $x \in A^c$ by defn of intersection

$\therefore$ $x \in A$ and $x \notin A$ by defn of complement

this is a contradiction!

$\therefore$ our assumption is incorrect, thus we can conclude $A \cap A^c = \emptyset$

3. Disprove if $A \not\subseteq B$ and $B \not\subseteq C$ then $A \not\subseteq C$




$A = \{1\}$, $C = \{1\}$, $B = \{2\}$

18. Disprove that for all sets A and B,

$$P(A \cup B) \subseteq P(A) \cup P(B)$$

$A = \{1\}$ $\quad P(A) = \{\emptyset, \{1\}\}$

$B = \{2\}$ $\quad P(B) = \{\emptyset, \{2\}\}$

$$P(A \cup B) = P(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$$

$$P(A) \cup P(B) = \{\emptyset, \{1\}, \{2\}\}$$

31. Construct an algebraic proof to show that for all sets A and B, $A \cup (B - A) = A \cup B$

$A \cup (B - A)$ starting point (given)

$= A \cup (B \cap A^c)$ set difference defn

$= (A \cup B) \cap (A \cup A^c)$ distributive law

$= (A \cup B) \cap U$ universal set defn

$= (A \cup B)$ identity property

$\therefore A \cup (B - A) = A \cup B$