SAMPLE SOLUTION

CSCI 246, Quiz 3 – Friday, April 1, 2022

Question One. 20 points. Consider the following two python functions:

```
def f(n):                    # n is a real number
    return 3*n + 7

def g(n):                    # n is a real number
    return n*n – 2
```

(a) 5 points. Find (f °g)(4.0) $f(g(4)) = f(14) = 49$

(b) 5 points. Find (g °f)(4.0) $g(f(4)) = g(19) = 19^2 - 2 = 359$

(c) 10 points. Write a python function named **f_inverse** such that (f °f_inverse)(x) = (f _inverse°f)(x) = x for all real numbers x.

```
def f_inverse(n):
    return (n-7)/3
```

Question Two. 20 points. Define F: **Z** → **Z** by the rule F(n) = 2 – 3n. Is F onto? Prove or give a counterexample.

not onto

Consider 0 — for $F(n) = 0$, we need to find n such that

$0 = 2 - 3n$

or $-2 = -3n$

or $2 = 3n$

or $\frac{2}{3} = n$

but $\frac{2}{3} \notin \mathbb{Z}$ ∴ F is not onto $\mathbb{Z}$

Question Three. 20 points. Let X = {a, b, c} and let P(X) be the power set of X. A relation **R** is defined on P(X) as follows: For every B, C ∈ P(X),

B **R** C ←→ the number of elements of B is less than the number of elements of C.

(a) 5 points. Is **R** reflexive? Explain briefly.

no – the size of a set is not less than the size of itself

e.g. $|A| \not< |A|$ $\forall A \in P(X)$

(b) 5 points. Is **R** symmetric? Explain briefly.

no – $\forall B, C \in P(X)$ if B R C then $|B| < |C|$

however, it can not be the case that C R B or $|C| < |B|$

(c) 5 points. Is **R** transitive? Explain briefly.

yes – $\forall A, B, C \in P(X)$ if A R B and B R C

that means $|A| < |B|$ and $|B| < |C|$

by transitivity $|A| < |C|$ or alternatively A R C

(d) 5 points. Is **R** an equivalence relation? Explain briefly.

no – it's neither reflexive nor symmetric

Question Four. 20 points. Consider the function h: $\mathbf{Q} \to \mathbf{Q}$ that is defined by the rule
$h(m/n) = m^2/n$ for integers m and n with $n \neq 0$.
Is h well defined? Justify your answer.

no - $\frac{1}{2} = \frac{2}{4}$

but $h\left(\frac{1}{2}\right) = \frac{1}{2}$

$h\left(\frac{2}{4}\right) = \frac{4}{4}$

since $h\left(\frac{1}{2}\right) \neq h\left(\frac{2}{4}\right)$,

h is not well defined

Question Five. 20 points. Construct an algebraic proof that for all sets A, B and C,

$$(A - B) - C = A - (B \cup C)$$

Cite a property from Theorem 6.2.2 for each step of the proof in the Justification column. Use as many steps as you need.

	Step in Proof	Justification
	$(A - B) - C$	Starting Point
$=$	$(A \cap B^c) - C$	set difference definition
$=$	$(A \cap B^c) \cap C^c$	" " "
$=$	$A \cap (B^c \cap C^c)$	associative property
$=$	$A \cap (B \cup C)^c$	DeMorgan's Law
$=$	$A - (B \cup C)$	set difference definition