CSCI 338, Second Midterm – March 9th, 2016

Name Sample Solution

Question One. 25 points. Consider language A, $A = \{a^{2n}b^{3n} \mid n >= 0\}$.

(a) 4 points. Write a context-free grammar (CFG) for A using as few rules as possible.

$S \rightarrow aaSbbb \mid \epsilon$

(b) 4 points. What is V in your CFG?

$\{S\}$

(c) 4 points. What is Σ in your CFG?

$\{a, b\}$

(d) 4 points. What is R in your CFG?

$\{S \rightarrow aaSbbb,\ S \rightarrow \epsilon\}$

(e) 4 points. What is S in your CFG?

S

(f) 5 points. Rewrite your CFG in Chomsky-Normal Form.

~~$S_0 \rightarrow S \mid \epsilon$~~

~~$S \rightarrow aaSbbb$~~

$S \rightarrow AC \mid \epsilon$

$C \rightarrow AD$

$D \rightarrow SE$

$E \rightarrow BF$

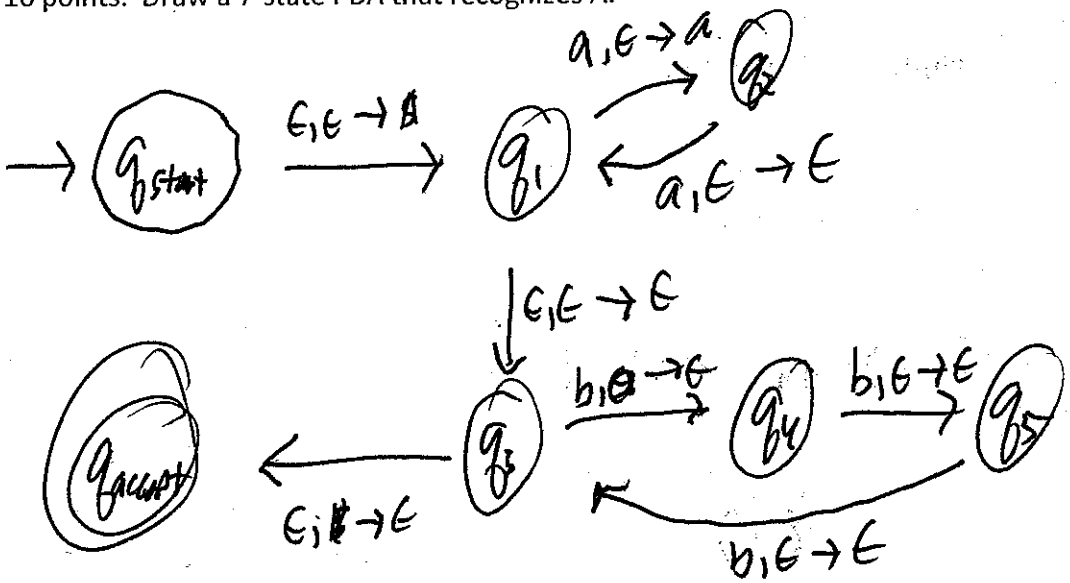
$F \rightarrow BB$

$A \rightarrow a$

$B \rightarrow b$

Question Two. 25 points. Consider language A from Question One.

(a) 10 points. Draw a 7-state PDA that recognizes A.



(b) 3 points. What is Q in your PDA?

$\{q_{start}, q_1, q_2, q_3, q_4, q_5, q_{loop}\}$

(c) 3 points. What is Σ in your PDA?

$\{a, b\}$

(d) 3 points. What is Γ in your PDA?

$\{a, \$\}$

(e) 3 points. What is q_0 in your PDA?

q_{start}

(f) 3 points. What is F in your PDA?

$\{q_{accept}\}$

Question Three. 25 points. Prove that language B = $\{a^{2^n} \mid n >= 0\}$ is not a context-free language.

Consider the string a^{2^p}

$uvxyz = a^{2^p}$ $\quad |uvxyz| = 2^p$

Since $|vxy| \leq p$ and $|vy| \geq 1$,

call $|vy|$ k

pump twice

$|uv^2xy^2z| = |uvxyz| + |vy|$

$= 2^p + k < 2^p + 2^p$

because $p < 2^p$

$\therefore uv^2xy^2z \notin B$

Question Four. 25 points.

(a) 5 points. True or False. Adding a second stack to a PDA increases the set of languages that can be recognized. If True, provide a language that can now be recognized. If False, briefly explain why the two are equivalent.

True. Consider $\{a^n b^n c^n \mid n \geq 0\}$

(b) 5 points. What is the key difference between a finite state automaton and a pushdown automaton?

PDA has a stack

(c) 5 points. Add one or more rules to the following CFG to generate the Kleene star of the original language. Do not change what either S or A generates.

$S_0 \rightarrow SS_0 \mid \epsilon$

$S \rightarrow aSa \mid bSb \mid A$
$A \rightarrow a \mid b$

(d) 5 points. True or False. The above CFG can be captured by a DPDA. Briefly explain.

False. Palindromes require guessing when to start the matching process

(e) Consider a PDA that has 10 states and accepts strings that contain x's, y's and z's. What is the maximum number of rules of the form $A_{pq} \rightarrow a\, A_{rs}\, b$ that can be produced? Briefly explain.

p q a r s b
10 · 10 · 4 · 10 · 10 · 4 = 160,000