Oceanographers develop a numerical simulation model of global ocean circulation. To obtain accurate results, the scientists divide the ocean into 4096 regions running east and west, and 1024 regions running north to south. In addition, they divide the ocean into 12 layers, with deeper layers containing sea water with greater density.

A single iteration of the model simulates ocean circulation for 10 minutes. One update requires 100 floating point operations.

How long will it take to simulate ocean circulation for a period of a year? Assume the supercomputer performs 1 Giga Flops.

\[
\text{Total no. of grids} = 1024 \times 4096 \times 12
\]

\[
\text{No. of calculation required for 1 grid for a single year} = 60 / 10 = 6.
\]

Assuming that there are 365 days in a year.

\[
\text{No. of calculations for a 1 grid for a single year} = 6 \times 24 \times 365
\]

\[
\text{Total no. of calculations required} = 1024 \times 4096 \times 12 \times 6 \times 24 \times 365
\]

Since each operation requires 100 fps.

\[
\text{Total fps required} = 1024 \times 4096 \times 12 \times 6 \times 24 \times 365 \times 100 \text{ fps}
\]

1 GFlops of a supercomputer means 1,000,000,000 fps.

\[
\text{Time req. for the computation} = \frac{1024 \times 4096 \times 12 \times 6 \times 24 \times 365 \times 100}{1,000,000,000,000}
\]

\[
\approx 264,540.14 \text{ seconds}
\]

\[
\approx 73.48 \text{ hours} \approx 3 \text{ days and 1.48 hours}
\]
5* [3] Given a data item originally available at a single processor in some model of parallel computation, let the function \( f(k) \) be the maximum number of processors to which the data can be transmitted in \( k \) or fewer data moving steps. (7 pts)

(1) Derive the formula for \( f(k) \) for the 2-D mesh.

\[
\text{for a hypercube } f(0) = 1, f(1) = 5, f(2) = 13, f(3) = 25.
\]

\[
\text{By induction } f(k) = 2k^2 + 2k + 1.
\]

(2) What are \( f(1) \) and \( f(2) \) for the hypercube of dimension 4?

Assuming a 2-ary hypercube of dim 4 \( \rightarrow f(1) = 5 \text{ & } f(2) = 11 \)

*(3) Derive the formula for \( f(d,k) \), \( k = 0, 1, \ldots, d \), for the hypercube of dimension \( d \).
From Pascal's Triangle:

$$\sum_{i=0}^{k} \frac{d!}{i! \, (d-i)!} \quad \sum_{i=0}^{k} \binom{d}{i}$$

$$f(d,k) = \begin{cases} f(d, k-1) + \binom{d}{k} \\ f(d, 0) = 1 \end{cases}$$