CSCI 246 – Class 13

SET PARTITIONS, FUNCTIONS
Quiz Questions

- Lecture 22:
  - Give an example of a partition of $A = \{1,2,3,4\}$

- Lecture 23:
  - What is $f^{-1}(y)$ for $y = ax+b$?
Midterm
Muffin Apology
Homework Extra Credit due on Friday
Defn: Two sets $A$ and $B$ have same cardinality iff there exists a function from $A$ to $B$ such that:

1. $F$ is **one to one**
Defn: Two sets A and B have same cardinality iff there exists a function from A to B such that:

1. F is one to one
2. F is onto

Set Partitions

- a collection of sets \( \{a_1, A_2, \} \) (finite or infinite) is a partition of set \( A \) iff:
  1. \( \bigcup_i A_i = A \)
Lesson 22 – Set Ids, Partitions

Set Partitions

- a collection of sets \( \{a_1, A_2, \} \) (finite or infinite) is a partition of set \( A \) iff:
  1. \( \bigcup_i A_i = A \)
  2. \( \forall i, j \text{ if } i \neq j, A_i \cap A_j = \emptyset \)
Lesson 22 – Set Ids, Partitions

Set Partitions

- a collection of sets \{a_1, A_2, \} (finite or infinite) is a partition of set A \iff:
  1. \( \bigcup_i A_i = A \)
  2. \( \forall i, j \text{ if } i \neq j, A_i \cap A_j = \emptyset \)

Consider Set \( A = \{1, 2, 3, 4, 5, 6\} \)
- A potential set partition for \( A \) is: \{\{1, 3\}, \{2\}, \{4, 5, 6\}\}
Lesson 23 - Functions

- Bijections
- Function Image of a set
- Inverse Image of a set
- Inverse Function
- Composing Functions
Lesson 23 - Functions

- Bijections
  - https://www.youtube.com/watch?v=dY9Lt lw-yP0
Not injective \[ \xrightarrow{3 \text{ got hit twice}} \]

Not surjective

Bijective = Surjective and injective

\[ \xrightarrow{\text{(everybody gets hit at least once) and (everybody that dies gets hit at most once)}} \]
Lesson 23 - Functions

- Function Image of a set
  - (informally) Subset of the Range which the domain maps to
Lesson 23 - Functions

- Inverse Image of a set
  - Aka: preimage
  - (informally) what values of X map to Y
Lesson 23 - Functions

- Inverse Function
  - Let \( f: X \to Y \) be one to one, onto function. we define \( f^{-1}: Y \to X \)
Lesson 23 - Functions

- Inverse Function
  - Let f: X -> Y be one to one, onto function. we define f⁻¹: Y -> X
  - Say y = 5x³, what’s f⁻¹(y)?
Lesson 23 - Functions

- Composing Functions
  - https://www.youtube.com/watch?v=jIlD_mIJIx4
\[ h(x) = 3x \]
\[ g(t) = -2t^2 - 2 - h(t) \]
\[ f(n) = -5n^2 + h(n) \]
\[ h(g(8)) = ? \]

\[ g(8) = -2 \cdot 8 - 2 - h \]
1. For the following functions, state whether they are one-to-one, onto, both, or neither
   
   a) \( f(x) = 3x + 4 \)
   
   b) \( f(x) = x^2 - 2 \)

2. Is \( \{A, C, D, Z\} \{B, X, D\} \) a set partition of \( \{A, B, C, D, X, Z\} \)? Why or why not?

3. Give \( f^{-1}(y) \) for the following: \( y = 3x^2 \cdot 9 - 2 \)

4. Given \( f(x) = 2x + 3 \), find \( (f \circ f)(x) \)

5. Given \( f(x) = 2x + 3 \), find \( (f \circ f)(5) \) and \( (f \circ f)(10) \)
Homework (Individual)

1. For the following functions, state whether they are one-to-one, onto, both, or neither
   a) \( f(x) = x^3 \)
   b) \( f(x) = |x - 2| \)


3. Give \( f^{-1}(y) \) for the following: \( y = 5x + 10 \)

4. Given \( f(x) = 2x + 3 \) and \( g(x) = -x^2 + 5 \), find \( (f \circ g)(x) \)

5. Given \( f(x) = 2x + 3 \) and \( g(x) = -x^2 + 5 \), find \( (g \circ f)(5) \) and \( (g \circ f)(10) \)