MORE FUNCTIONS
Quiz Questions

- Lecture 24:
  - If $f: X \to Y$ is one to one, and $g: Y \to Z$ is one to one, then $(gof)(x)$ is: __________
  - If $f: X \to Y$ is onto, and $g: Y \to Z$ is onto, then $(gof)(x)$ is: __________
  - Let $|X| = m$, and $|Y| = n$, what can we say about the relationship between $m$ and $n$ when:
    - $f$ is one to one?
    - $f$ is onto?
    - $f$ is one to one and onto?
midterms/homeworks (except 11) are graded
If $f: X \rightarrow Y$ is one to one, and $g: Y \rightarrow Z$ is one to one, then $(gof)(x)$ is one to one.

If $f: X \rightarrow Y$ is onto, and $g: Y \rightarrow Z$ is onto, then $(gof)(x)$ is onto.

Onto Proof: https://www.youtube.com/watch?v=i5b4Mr8Aa4A
Prove that if \( g: A \rightarrow B \) and \( f: B \rightarrow C \) are onto, then \( f \circ g: A \rightarrow C \) is onto.

Proof (Recall \( f: X \rightarrow Y \) is onto if \( \forall y \in Y \exists x \in X \) s.t. \( f(x) = y \)).

Suppose \( c \in C \) is arbitrary. Since \( f \) is onto, \( \exists b \in B \) s.t. \( f(b) = c \).

Since \( g \) is onto, \( \exists a \in A \) s.t. \( g(a) = b \).

Then \( (f \circ g)(a) = f(g(a)) = f(b) = c \). Since \( c \in C \) was arbitrary, \( \forall c \in C \), \( (f \circ g)(a) = c \). Thus \( \forall c \in C \), \( (f \circ g)(a) = c \). Therefore, \( f \circ g \) is onto.
Let $|X| = m$, and $|Y| = n$, what can we say about the relationship between $m$ and $n$ when:

- $f$ is one to one?
Let $|X| = m$, and $|Y| = n$, what can we say about the relationship between m and n when:

- f is one to one?

One to one: For every point in $x$, there must be a distinct point in $y$. 
Lesson 24

- Let $|X| = m$, and $|Y| = n$, what can we say about the relationship between $m$ and $n$ when:
  - $f$ is one to one?

One to one: For every point in $x$, there must be a distinct point in $y$

$m \leq n$
Let $|X| = m$, and $|Y| = n$, what can we say about the relationship between $m$ and $n$ when:

- $f$ is one to one?

One to one: For every point in $x$, there must be a distinct point in $y$

$m \leq n$
Let $|X| = m$, and $|Y| = n$, what can we say about the relationship between $m$ and $n$ when:
- $f$ is onto?
Let $|X| = m$, and $|Y| = n$, what can we say about the relationship between $m$ and $n$ when:

- $f$ is onto?

**Onto:** All elements in $y$ are used
Let $|X| = m$ and $|Y| = n$, what can we say about the relationship between $m$ and $n$ when:

- $f$ is onto?

Onto: All elements in $Y$ are used

$m > n$
Let $|X| = m$, and $|Y| = n$, what can we say about the relationship between $m$ and $n$ when:

- $f$ is onto?

Onto: All elements in $Y$ are used

$m > n$
Lesson 24

Let $|X| = m$, and $|Y| = n$, what can we say about the relationship between $m$ and $n$ when:
- $f$ is onto?

Onto: All elements in $Y$ are used

$m > n$
Let $|X| = m$, and $|Y| = n$, what can we say about the relationship between $m$ and $n$ when:

- $f$ is onto?

**Onto:** All elements in $Y$ are used

$m > n$
Lesson 24

- Pigeon Hole:

  - https://www.youtube.com/watch?v=ROnetLvbl6M
PIGEONHOLE PRINCIPLE
Homework (Group)

1. **let**: $f(x) = -4x + 9$ and $g(x) = 2x - 7$, Find $(f \circ g)(x)$, is it one-to-one, onto, or both?

2. **let**: $h(x) = 3x - 5$ and $g(x) = 2x^2 - 7x$, find $(h \circ g)(x)$ is it one-to-one, onto, or both?

3. **let**: $f(x) = -4x + 9$, $g(x) = 2x - 7$, $h(x) = 3x - 5$, Find $(f \circ g \circ h)(x)$, is it one-to-one, onto, or both?

4. Say size of set $A = 50$ and size of Set $B = 55$, what can we say about a possible relationship between $A$ and $B$?

5. Say size of set $A = 5$ and size of Set $B = 5$, what can we say about a possible relationship between $A$ and $B$?
1. Let: \( f(x) = -4x + 9 \) and \( g(x) = 2x - 7 \), find \((g \circ f)(x)\), is it one-to-one, onto, or both?

2. Let: \( h(x) = 3x - 5 \) and \( g(x) = 2x^2 - 7x \), find \((g \circ h)(x)\), is it one-to-one, onto, or both?

3. Let: \( f(x) = -4x + 9 \), \( g(x) = 2x - 7 \), \( h(x) = 3x - 5 \), find \((h \circ f \circ g)(x)\), is it one-to-one, onto, or both?

4. Say size of set \( A = 56 \) and size of Set \( B = 55 \), what can we say about a possible relationship from \( A \) to \( B \)?

5. Say size of set \( A = 13 \) and size of Set \( B = 13 \), what can we say about a possible relationship from \( A \) to \( B \)?