Lecture 11:
- Remember the factorization of n=10? Give the following values:
  - $P_1 = ?$ $e_1 = ?$
  - $P_2 = ?$ $e_2 = ?$

Lecture 12:
- What is the GCD of 20 and 30?
Late Policy Update
Class Slides posted to website
Quiz tomorrow: Covers from Last Friday (proofs) to Today
Submit questions by 5pm for extra credit
Quotients q vs b (from Quotient remainder Theorem) vocab:

- “Divide 32 by 5"
- Dividend = Divisor x Quotient + Remainder
- \[ 32 = 5 \times 6 + 2 \]
#5 from group homework yesterday:

Show that if $a | b$ and $b | a$, where $a$ and $b$ are integers, then $a=b$ or $a=-b$

$$a = bd$$

$$b = ac$$

$$a = acd, \text{ thus } c = d = 1 \text{ or } c = d = -1$$
#5 from group homework yesterday:

Show that if \( a \mid b \) and \( b \mid a \), where \( a \) and \( b \) are integers, then \( a=b \) or \( a=-b \)

\[
\begin{align*}
a &= bd \\
b &= ac \\
a &= acd, \text{ thus } c = d = 1 \text{ or } c = d = -1
\end{align*}
\]

Because this is asking to show and not prove, it’s legitimate to give examples (as a reminder, there is no such thing as proof by example).

Consider \( a = 8 \), and \( b = -8 \) or \( a = -8 \) and \( b = 8 \),
Lesson 11

Unique Factorization (aka Fundamental Theorem of Arithmetic) Theorem

- Given any integer $n > 1$, there exist prime numbers $(P_1 > P_2 > \cdots > P_k)$ and positive integers $(e_1, e_2, \ldots, e_k)$ such that $n = P_1^{e_1} P_2^{e_2} \cdots P_k^{e_k}$
Lesson 11

¬ Unique Factorization (aka Fundamental Theorem of Arithmetic) Theorem

¬ Given any integer $n > 1$, $\exists$ prime numbers $(P_1 > P_2 > \cdots > P_k)$and positive integers $(e_1, e_2, \ldots, e_k)$

such that $n = P_1^{e_1}, P_2^{e_2}, \ldots P_k^{e_k}$

e.g. $n = 10$
Lesson 11

- Unique Factorization (aka Fundamental Theorem of Arithmetic) Theorem
  - Given any integer $n > 1$, $\exists$ prime numbers $(P_1 > P_2 > \cdots > P_k)$ and positive integers $(e_1, e_2, \ldots, e_k)$ such that $n = P_1^{e_1} P_2^{e_2} \cdots P_k^{e_k}$

  e.g. $n = 10 = 5 \times 2$
Lesson 11

- Unique Factorization (aka Fundamental Theorem of Arithmetic) Theorem
  - Given any integer \( n > 1 \), \( \exists \) prime numbers \( (P_1 > P_2 > \cdots > P_k) \) and positive integers \( (e_1, e_2, \ldots, e_k) \)
    
    such that \( n = P_1^{e_1} \cdot P_2^{e_2} \cdots P_k^{e_k} \)

  - e.g. \( n = 10 = 5^1 \cdot 2^1 \)
Unique Factorization (aka Fundamental Theorem of Arithmetic) Theorem

Given any integer $n > 1$, there exist prime numbers $(P_1 > P_2 > \cdots > P_k)$ and positive integers $(e_1, e_2, \ldots, e_k)$ such that $n = P_1^{e_1} \cdot P_2^{e_2} \cdots P_k^{e_k}$

e.g. $n = 10 = 5^1 \cdot 2^1$

$P_1 = 5, \ e_1=1$

$P_2 = 2, \ e_2=1$
Lesson 11

- Unique Factorization (aka Fundamental Theorem of Arithmetic) Theorem
  - Given any integer \( n > 1 \), \( \exists \) prime numbers \( (P_1 > P_2 > \cdots > P_k) \) and positive integers \( (e_1, e_2, \ldots, e_k) \) such that \( n = P_1^{e_1} \times P_2^{e_2} \times \cdots \times P_k^{e_k} \).

  e.g. \( n = 10 = 5^1 \times 2^1 \)
  
  \( P_1 = 5, \; e_1 = 1 \)
  
  \( P_2 = 2, \; e_2 = 1 \)
Lesson 11

Unique Factorization (aka Fundamental Theorem of Arithmetic) Theorem

- Given any integer $n > 1$, there exist prime numbers $(P_1 > P_2 > \cdots > P_k)$ and positive integers $(e_1, e_2, \ldots, e_k)$ such that $n = P_1^{e_1} \cdot P_2^{e_2} \cdot \cdots \cdot P_k^{e_k}$

For example, show factorization for $n = 98$
Lesson 11

- Irrational Numbers
  - \( \overline{\mathbb{Q}} = \mathbb{R} - \mathbb{Q} = \{ x \in \mathbb{R} | x \notin \mathbb{Q} \} \)
Lesson 11

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Lesson 11

- Irrational Numbers
  - $\bar{\mathbb{Q}} = \mathbb{R} - \mathbb{Q} = \{x \in \mathbb{R} | x \notin \mathbb{Q}\}$

- Say $S1 = \{1,2,3,4,5\}$ and $S2 = \{1,2,3\}$, what is $S1 - S2$?
Lesson 11

- Irrational Numbers
  - $\overline{\mathbb{Q}} = \mathbb{R} - \mathbb{Q} = \{x \in \mathbb{R} | x \notin \mathbb{Q}\}$

- Say $S_1 = \{1, 2, 3, 4, 5\}$ and $S_2 = \{1, 2, 3\}$, what is $S_1 - S_2$?

- What if $S_1 = \{1, 2, 3, 4, 5\}$ and $S_2 = \{-5, -3, -1, 1, 3, 5\}$, what is $S_1 - S_2$?
Lesson 12

- Contrapositive
  - **Note**: It was not required to use contraposition to prove that if \( n^2 \) is even then \( n \) is even. You can use direct proof to show this.

- Prove that if \( n^2 \) is even \( n \) is even by proving that if \( n \) is not even (i.e. negative) then \( n \) is not even (i.e. odd)
  - **Note**: Whenever you want to prove something is odd, remember that odd can be thought of as \( n=2k+1 \).
  - \( n^2=(2k+1)(2k+1)=4k^2+4k+1 \)
Lesson 12

- Greatest Common Factor (GCD)
  - Consider GCD(a,b)
    - 1. find all the divisors of a
    - 2. find all the divisors of b
    - 3. Find the greatest divisor that both and be have in common – may be 1
1. Find the prime factorization of the following:
   a) 100
   b) 999
   c) 1024

2. Use proof by contradiction to show that there are infinitely many prime numbers (hint, remember the unique factorization theorem)

3. For the following, give the Greatest Common Divisor (GCD):
   a) GCD(24, 36)
   b) GCD(17, 22)
1. Give the prime factorization of the following:
   a) 88
   b) 126
   c) 9

2. Give the Greatest Common Divisor for the following:
   a) GCD(21, 28)
   b) GCD(90, 135)
   c) GCD(63, 42)

3. Let $S_1 = \{x | x \in \text{Hours on a standard clock}\}$ and $S_2 = \{z | z \in \mathbb{Z} \land z > 0\}$, what is $S_1 - S_2$?