Lecture 17:
- Prove $P(n): 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

Lecture 18:
- Geometric Recursion: $a_n = ?$
- Arithmetic Recursion: $a_n = ?$
Notes

- Don’t forget □ at the end of proofs
- Content is getting harder, don’t panic
- This Friday:
  - Midterm: 1 sheet of notes, calculators allowed, resources will be updated on website.
  - Course Mini-eval
  - Project
Lesson 17 - (Principal) Mathematical Induction

https://www.youtube.com/watch?v=wblW_M_HVQ8
Prove by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for every positive integer $n$. 
Prove by induction that \(1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}\) for every positive integer \(n\).

**Basis:** When \(n = 1\)
- Left hand side of the equation is 1
- Right hand side is \(\frac{1(1+1)(2*1+1)}{6} = \frac{2*3}{6} = 1\). So \(P(1)\) is correct.
Prove by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for every positive integer $n$.

**Induction hypothesis:** Assume that $P(k)$ is correct for some positive integer $k$. That means that the left hand side of the equation equals the right hand side, so $1^2 + 2^2 + 3^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$.
Lesson 17 - (Principal) Mathematical Induction

Prove by induction that \(1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}\) for every positive integer \(n\).

**Induction:**
- \(1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2\) (left side)
Prove by induction that \( 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \) for every positive integer \( n \)

**Induction:**
- \( 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 \) (left side)
- \( = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \) (by our induction hypothesis)
Lesson 17 - (Principal) Mathematical Induction

- Prove by induction that \( 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \) for every positive integer \( n \)

- **Induction:**
  - \( 1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2 \) (left side)
  - \[ = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \] (by our induction hypothesis)
  - \[ = \frac{k(k+1)(2k+1)}{6} + \frac{6*(k+1)^2}{6} \]
Lesson 17 - (Principal) Mathematical Induction

- Prove by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for every positive integer $n$

- Induction:
  - $1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2$ (left side)
  - $= \frac{k(k+1)(2k+1)}{6} + (k + 1)^2$ (by our induction hypothesis)
  - $= \frac{k(k+1)(2k+1)}{6} + \frac{6 * (k+1)^2}{6}$
  - $= \frac{k(k+1)(2k+1) + 6 * (k+1)^2}{6}$
Prove by induction that \(1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}\) for every positive integer \(n\).

**Induction:**

- 1\(^2\) + 2\(^2\) + 3\(^2\) + \ldots + k\(^2\) + (k+1)\(^2\) (left side)

\[
= \frac{k(k+1)(2k+1)}{6} + (k + 1)^2 \quad \text{(by our induction hypothesis)}
\]

\[
= \frac{k(k+1)(2k+1)}{6} + \frac{6 \cdot (k+1)^2}{6}
\]

\[
= \frac{k(k+1)(2k+1) + 6 \cdot (k+1)^2}{6}
\]

\[
= \frac{(k+1)[(2k+1)+ 6 \cdot (k+1)]}{6}
\]
Prove by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for every positive integer $n$.

**Induction:**
1. $1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2$ (left side)
2. 
   
   $= (k+1)[(2k+1) + 6 \cdot (k+1)]$

   $= \frac{(k+1)(2k^2 + 7k + 6)}{6}$

Lesson 17 - (Principal) Mathematical Induction

Prove by induction that \(1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}\) for every positive integer \(n\)

**Induction:**

- \(1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2\) (left side)

\[
\begin{align*}
\Rightarrow & \frac{(k+1)[(2k+1)+ 6 *(k+1)]}{6} \\
\Rightarrow & \frac{(k+1)(2k^2 + 7k + 6)}{6} \\
\Rightarrow & \frac{(k+1)(k+2)(2k+3)}{6}
\end{align*}
\]
Prove by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for every positive integer $n$.

**Induction:**

1. **Left Side:**
   
   $1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2$  

2. **Right Side:**
   
   
   $\frac{(k+1)[(2k+1)+ 6 *(K+1)]}{6}$  

   $\frac{(k+1)(2k^2 + 7k + 6)}{6}$  

   $\frac{(k+1)(k+2)(k+3)}{6}$  

   $\frac{(k + 1)((k + 1) + 1)(2(k + 1) + 1)}{6}$
Prove by induction that \(1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}\) for every positive integer \(n\).

**Induction:**

- Left side: \(1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2\)

\[
\begin{align*}
\text{Right side} & = \frac{(k+1)[(2k+1) + 6 *(K+1)]}{6} \\
& = \frac{(k+1)(2k^2 + 7k + 6)}{6} \\
& = \frac{(k+1)(k+2)(2k+3)}{6} \\
& = \frac{(k + 1)((k + 1) + 1)(2(k + 1) + 1)}{6} \quad \text{(Right Hand Side)}
\end{align*}
\]
Prove by induction that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for every positive integer $n$.

**Induction:**
- Left side: $1^2 + 2^2 + 3^2 + \ldots + k^2 + (k+1)^2$
- \[= \frac{(k+1)((2k+1)+6*(k+1))}{6}\]
- \[= \frac{(k+1)(2k^2 + 7k + 6)}{6}\]
- \[= \frac{(k+1)(k+2)(2k+3)}{6}\]
- \[= \frac{(k + 1)((k + 1) + 1)(2(k + 1) + 1)}{6}\] (Right Hand Side)

**By induction**, $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$
In mathematics, a recurrence relation is an equation that recursively defines a sequence. Once one or more initial terms are given: each further term of the sequence or array is defined as a function of the preceding terms.

-Wikipedia
Lesson 18 – Recurrence Relations

- Towers of Hanoi

1 disk: 1 move

Move 1: move disk 1 to post C

2 disks: 3 moves

Move 1: move disk 2 to post B
Move 2: move disk 1 to post C
Move 3: move disk 2 to post C
Lesson 18 – Recurrence Relations

3 disks: 7 moves

Move 1: move disk 3 to post C
Move 2: move disk 2 to post B
Move 3: move disk 3 to post B
Move 4: move disk 1 to post C
Move 5: move disk 3 to post A
Move 6: move disk 2 to post C
Move 7: move disk 3 to post C
Lesson 18 – Recurrence Relations

- Towers of Hanoi
1. For \( n > 1 \), prove that by induction \( 1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + (n)(n+1) = \frac{(n)(n+1)(n+2)}{3} \)

2. Find the first five terms for each of the following
   a) \( a_n = a_{n-1} \cdot a_0 = 2 \)
   b) \( a_n = a^{2}_{n-1} \cdot a_0 = 2 \)
   c) \( a_n = a_{n-1} + 3 \cdot a_{n-2} \cdot a_0 = 2 \)
1. Prove by induction that $11^n - 6$ is divisible by 5 for every positive integer $n$.

2. Find the first five terms for each of the following
   a) $a_n = n*a_{n-1} + n^2*a_{n-2}$, $a_0 = 1$, $a_1 = 1$
   b) $a_n = a_{n-1} + a_{n-3}$, $a_0 = 1$, $a_1 = 2$, $a_2 = 0$