Quiz Questions

- Lecture 14:
  - What is the Big-O notation for Euclid's Algorithm runtime?

- Lecture 15:
  - Show $f(n) = 10n + 5$ is in $O(n^2)$
Notes

- Modified Office Hours Today
- Midterm
- Homework Handback
Lesson 14 & 15

- Big – O => Upper Bound
Big – O => Upper Bound

\[ f(n) = O(g(n)) \text{ if there exist positive constants } c \text{ and } n_0 \text{ such that } f(n) \leq c \cdot g(n) \text{ for all } n \geq n_0 \]
Lesson 14 & 15

- **Big – O => Upper Bound**
  - $f(n) = O(g(n))$ if there exist positive constants $c$ and $n_0$ such that $f(n) \leq c \cdot g(n)$ for all $n \geq n_0$

- **Example:** ([http://www.fas.harvard.edu/~cscie119/lectures-sorting.pdf](http://www.fas.harvard.edu/~cscie119/lectures-sorting.pdf))
  - $f(n) = n^2/2 - n/2$ is $O(n^2)$, because $n^2/2 - n/2 \leq n^2$ for all $n \geq 0$
Lesson 14 & 15

- **Big – O => Upper Bound**
  - $f(n) = O(g(n))$ if there exist positive constants $c$ and $n_0$ such that $f(n) \leq c \cdot g(n)$ for all $n \geq n_0$

- **Example**: ([http://www.fas.harvard.edu/~cscie119/lectures/sorting.pdf](http://www.fas.harvard.edu/~cscie119/lectures/sorting.pdf))
  - $f(n) = \frac{n^2}{2} - \frac{n}{2}$ is $O(n^2)$, because $\frac{n^2}{2} - \frac{n}{2} \leq n^2$ for all $n \geq 0$

  - $c = 1$
  - $n_0 = 0$
Lesson 14 & 15

- **Big – \( \Omega \) => Lower Bound
- \( f(n) = O(g(n)) \) if there exist positive constants \( c \) and \( n_0 \) such that
  \[ f(n) \leq c \cdot g(n) \] for all \( n \geq n_0 \)
Lesson 14 & 15

- **Big – \( \Omega \) => Lower Bound**
- \( f(n) = \Omega(g(n)) \) if there exist positive constants \( c \) and \( n_0 \) such that
  \[
  f(n) \leq c \cdot g(n) \text{ for all } n \geq n_0
  \]

- **Example:**
  - \( n^3 + 4n^2 = \Omega(n^2) \) is \( O(n^2) \), because \( n^3 \leq n^3 + 4n^2 \) for all \( n \geq 0 \)
Big – $\Omega$ => Lower Bound

- $f(n) = \Omega(g(n))$ if there exist positive constants $c$ and $n_0$ such that $f(n) \leq c \times g(n)$ for all $n \geq n_0$

Example:

- $n^3 + 4n^2 = \Omega(n^2)$ is $\Omega(O(n^2))$, because $n^3 \leq c \times n^3 + 4n^2$ for all $n \geq 0$

```
c=1
no=0
```
Lesson 14 & 15

- **Big – Θ => Tight Bound**
  
  - $f(n) = \Theta(g(n))$ if there exists constants $c_1, c_2, c_3$ and $n_0$ such that
  
  $$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n) \text{ for all } n > n_0$$
Lesson 14 & 15

- **Big – Θ => Tight Bound**
  - \( f(n) = \Theta(g(n)) \) if there exists constants \( c_1, c_2, c_3 \) and \( n_0 \) such that
    - \( c_1g(n) \leq f(n) \leq c_2g(n) \) for all \( n > n_0 \)
  - **Example:** ([http://www.fas.harvard.edu/~cscie119/lectures/sorting.pdf](http://www.fas.harvard.edu/~cscie119/lectures/sorting.pdf))
  - \( f(n) = n^2/2 - n/2 \) is \( \Theta(n^2) \) because \( (1/4)n^2 \leq n^2/2 - n/2 \leq n^2 \) for all \( n \geq 2 \)
Big – $\Theta$ => Tight Bound

- $f(n) = \Theta(g(n))$ if there exists constants $c_1$, $c_2$, $c_3$ and $n_o$ such that
  - $c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n > n_o$

- **Example**: (http://www.fas.harvard.edu/~cscie119/lectures/sorting.pdf)

- $f(n) = n^2/2 - n/2$ is $\Theta(n^2)$ because $(1/4) n^2 \leq n^2/2 - n/2 \leq n^2$ for all $n \geq 2$

  - $c_1 = 1/4$
  - $c_2 = 1$
  - $n_o = 2$
Lesson 14 & 15
Determine whether each of these functions is $O(x)$ by giving a $C$ and $n_0$ value where appropriate

1. $f(x) = 10$
2. $f(x) = 3x + 7$
3. $f(x) = x^2 + x + 1$
4. $f(x) = 5 \log x$
5. $f(x) = \text{floor}(x)$
6. $f(x) = \text{ceiling}(x)$
Homework (Individual)

1. Explain the differences between Big-O, Big-Ω (Omega), and Big-Θ (Theta)
2. Explains what it means for a function to be O(1) instead of O(n)
3. Show that $f(x) = (x + 5) \log_2 (3x^2 + 7)$ is $O(x \log_2 x)$
   a) Hint: Remember that $\log(x^k) = k \cdot \log(x)$
4. Consider: $f(x) = 15n^3 + n^2 + 4$,
   a) Express $f(x)$ in Big - O notation
   b) Express $f(x)$ in Big – Ω notation
   c) Express $f(x)$ in Big – Θ notation