A hidden Markov model for modelling long-term persistence in multi-site rainfall time series. 2. Real data analysis

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Abstract

The hidden Markov model (HMM) provides an attractive framework for modelling long-term persistence in hydrological data because it can produce time series with long-term wet and dry periods. In this study, the Bayesian calibration procedure for the multi-site HMM developed by Thyer and Kuczera [J. Hydrol. (2003)] is used to calibrate the model to multi-site rainfall data from the Warragamba, Central Coast and Williams River catchment regions—all important water supply catchments located on the east coast of Australia. This methodology is used to verify the majority of the HMM assumptions. The results for the Warragamba and Central Coast catchment region provided strong evidence that a model with a two-state persistence structure was more consistent with the data than a one state model with no persistence. These findings may have considerable implications for water resources management and drought risk assessment in both these regions. In addition, the results suggested that the multi-site framework exploits space-for-time substitution and the sampling of missing data to better identify the long-term persistence structure. For the Williams River catchment rainfall data difficulties were experienced with achieving convergence of the calibration procedure because of bimodal posterior distributions. While the results suggested a two-state persistence structure exists for the Williams, the difficulties indicate there is scope for further refinement of the implementation of the HMM concept.

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1. Introduction

Long-term persistent wet and dry periods are often present in long-term hydrological data due to the influence of quasi-cyclic climatic processes such as El Niño and the interdecadal Pacific oscillation. Thyer and Kuczera (2000) applied the single site hidden Markov model (HMM) to stochastically model long-term persistence in hydro-climatic time series because it could reproduce these wet and dry periods. The calibration results from the single site HMM supported the notion that it provided a conceptually sounder approach than other commonly used stochastic models (Thyer and Kuczera, 2000).
A Bayesian approach for calibrating a multi-site HMM to long-term rainfall time series was presented by Thyer and Kuczera (2003). The application of a multi-site HMM for modelling long-term persistence in multi-site hydrological time series was motivated by the following: (1) In large multi-reservoir water resource systems multi-site simulations of hydrological inputs are required. (2) With multi-site data the benefits of space-for-time substitution and the ability to handle missing data could potentially lead to better identification of the long-term persistence structure.

In this paper, this Bayesian procedure will be used to calibrate the multi-site HMM to real rainfall data from several important water supply catchments: the Warragamba and Central Coast catchment regions (both located near Sydney, Australia) and the Williams River catchment region, located in the lower Hunter region on the east coast of Australia.

The primary objective of this study is to examine if there is evidence of a long-term persistence structure in the rainfall regime over these water supply catchment regions rather than at a single site as was done by Thyer and Kuczera (2000). One would expect that given the large spatial extent of climate phenomena associated with global circulations such as El Niño and the interdecadal Pacific oscillation a homogenous persistence structure would exist over a large region. The secondary objectives are to determine whether multi-site data enhance the identification of the long-term regional persistence structure and to assess the influence of missing data.

This paper is organised as follows: in Section 2 the multi-site HMM is briefly outlined, including a summary of the modelling assumptions. Section 3 provides the relevant details of the rainfall data, followed by a short outline of the model calibration procedure in Section 4. The calibration results are then analysed in Section 5 to determine if each of the HMM assumptions are justified and if a strong persistence was identified. The discussion in Section 6 considers several issues related to the application of the multi-site HMM framework. These include the influence of multi-site data on the identification of a regional persistence structure, the influence of missing data, the development of a suitable methodology for identifying a homogenous persistence region, and the potential implications of applying the HMM framework.

2. Multi-site hidden Markov model

The multi-site HMM is fully explained by Thyer and Kuczera (2003), as a result only a brief outline will be given here. The framework of the HMM (Fig. 1) assumes the climate is one of two states; wet (W) or dry (D). The persistence in each state is governed by the state transition probabilities, $P(\text{Wet} \to \text{Dry})$ and $P(\text{Dry} \to \text{Wet})$, hereafter denoted as $p_{WD}$ and $p_{DW}$ respectively. Each state has an independent multi-variate Gaussian rainfall distribution, such that the vector of rainfall values, $y_t$ at $r$ multiple sites can be simulated using

$$y_t \sim N_r(\mu_k, \Sigma_k)$$

where $k$ is the climate state at time $t$, denoted by $s_t \in \{W, D\}$ and $N_r(\mu, \Sigma)$ denotes a multi-variate Gaussian distribution in $r$ dimensions, with mean vector $\mu$ and covariance matrix, $\Sigma$.

The vector of unknown parameters for the multi-site HMM, $\theta$ is

$$\theta' = (\mu_W, \Sigma_W, \mu_D, \Sigma_D, p_{WD}, p_{DW}, s_N)$$

where $s_N = \{s_1, s_2, ..., s_N\}$, the hidden state time series, is included with the model parameters because prior to model calibration it is unknown and must be estimated as part of the calibration process.

In summary, the following assumptions are made in calibrating the multi-site HMM to multi-site rainfall time series:

1. The distribution of rainfall data is composed of two independent distributions: a wet state and a dry state distribution.
2. Both the wet and dry state distributions are multi-variate Gaussian.
3. The climate state at time $t$, $s_t$ is purely dependent on the climate state from the previous time step, $s_{t-1}$.

Fig. 1. Model framework of the HMM.
4. The probability of state transition is assumed to be stationary over time.
5. Every site is assumed to be in the same climate state at every point in time, i.e. the climate state is assumed to be ‘regional’.

3. Model calibration—Gibbs sampler

To calibrate the multi-site HMM a Markov chain Monte Carlo (MCMC) method known as the Gibbs sampler is used. This Bayesian technique fully evaluates parameter uncertainty by inferring the probability distribution of the model parameters $\theta$ for the observed time series data $Y_{\text{obs}}^{\text{N}}$, referred to as the posterior distribution of the model parameters. In the presence of missing data, $Y_{\text{mis}}^{\text{N}}$, it is denoted as $p(\theta | Y_{\text{mis}}^{\text{N}})$. In the companion paper, Thyer and Kuczera (2003) provide a full description of the various challenges confronted in the implementation of the Gibbs sampling algorithm, primarily concerning the specification of appropriate priors. Also an outline of the methodology for handling missing data which enabled the analysis of noncontiguous data sets was given. In this study this methodology will be used to calibrate the multi-site HMM to the rainfall data described below.

4. Rainfall data

A major objective of this study is to determine if there exists a long-term regional persistence in the rainfall regimes of the Warragamba, Central Coast and the Williams River catchment regions. All these catchments are important water supply sources. The identification of a strong persistence structure could have serious ramifications for water resources management, particularly drought security, for these regions.

4.1. Warragamba catchment region

The Warragamba catchment region is located on the east coast of Australia (Fig. 2(a)). Its reservoir is an important water supply source, supplying approximately 80% of the water for Sydney’s population of almost 4 million people. The rainfall records used in this study were chosen from the numerous sites available within the Warragamba catchment region. Sites were selected based on the following criteria: there exists a long record and the rainfall regime in the catchment is evenly represented, given the available data. In the companion paper (Thyer and Kuczera, 2003) it was shown using synthetic data that the inclusion of highly correlated sites may not provide any additional rainfall information and furthermore may be detrimental to the performance of the Gibbs sampler. Therefore, if several sites were located close together and their rainfall data showed a very high correlation, only one site was chosen.

To represent the rainfall regime of the Warragamba catchment, data from six gauges was chosen and used to construct four individual time series. Two of the time series, Moss Vale and Taralga are the original rainfall records from their respective sites. The other two time series are composite records constructed when data from two nearby sites with a high correlation could be combined to produce an extended rainfall time series. The Mt. Victoria gauge was extended from 11/1990 to 6/1994 using the Blackheath record ($r^2 = 0.93$ for monthly rainfall) to form the Mt. Victoria composite series. Similarly, the Yarra record was extended back from 8/1896 to 9/1870 using the Goulburn gauge ($r^2 = 0.8$ for monthly values) to form the Yarra composite series. These four rainfall records were checked to ensure there were no gross inhomogeneities or inconsistencies. Data from an additional rain gauge with a shorter record, Cataract dam, was also available from the high quality data set produced by Lavery et al. (1997). Cataract dam is located in one of four smaller catchments that also form part of Sydney’s water supply and are located nearby to the Warragamba catchment (refer Fig. 2). A summary of the details of the five rainfall time series from the Warragamba dam catchment is given in Table 1. Fig. 2(a) shows the relative location of each of the original rainfall data sites.

4.2. Central Coast catchment region

The Central Coast catchment region is located approximately 50–100 km north of Sydney (Fig. 2(a)). Although it is considerably smaller than
the Warragamba catchment it is an important water supply source for approximately 300,000 people of the Gosford–Wyong region. Although there are no long-term continuous rainfall records actually located within the water supply catchment long-term records were available at Gosford and at Wyee which are both located within close proximity to the water supply catchment. It was decided to analyse these rainfall records to determine if there is any evidence of a two-state persistence structure within the Central Coast region.

![Fig. 2. Location of rainfall data sites.](image)

**Table 1**
Summary of monthly rainfall time series in Warragamba catchment region

<table>
<thead>
<tr>
<th>Time series</th>
<th>Start date</th>
<th>End date</th>
<th>Length (years)</th>
<th>Annual statisticsa</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Warragamba catchment region</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moss Vale</td>
<td>8/1870</td>
<td>6/1994</td>
<td>123</td>
<td>1003 (284)</td>
</tr>
<tr>
<td>Taralga</td>
<td>1/1882</td>
<td>7/1994</td>
<td>112</td>
<td>816 (229)</td>
</tr>
<tr>
<td>Yarra comp.</td>
<td>9/1870</td>
<td>6/1994</td>
<td>123</td>
<td>672 (191)</td>
</tr>
<tr>
<td>Cataract Dam</td>
<td>1/1904</td>
<td>12/1994</td>
<td>94</td>
<td>1076 (377)</td>
</tr>
<tr>
<td>(b) Central coast catchment region</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gosford</td>
<td>3/1885</td>
<td>7/1993</td>
<td>108</td>
<td>1297 (343)</td>
</tr>
<tr>
<td>Wyee</td>
<td>3/1899</td>
<td>5/1999</td>
<td>100</td>
<td>1192 (295)</td>
</tr>
<tr>
<td>(c) Williams River catchment region</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clarence Town PO</td>
<td>9/1895</td>
<td>4/1998</td>
<td>102</td>
<td>1060 (258)</td>
</tr>
<tr>
<td>Dungog PO</td>
<td>1/1897</td>
<td>11/1997</td>
<td>100</td>
<td>994 (239)</td>
</tr>
<tr>
<td>Raymond Terrace</td>
<td>4/1894</td>
<td>9/1997</td>
<td>103</td>
<td>1047 (259)</td>
</tr>
</tbody>
</table>

a For the annual statistics the empirical mean and standard deviation (in parentheses) are given.
A summary of the details of these two time series is provided in Table 1.

### 4.3. Williams River catchment region

The Williams River catchment is also located on the east coast of Australia, approximately 200–250 km north of the Warragamba dam catchment (Fig. 2(b)). The catchment supplies water for over 400,000 people in a region known as the Lower Hunter, on the east coast of New South Wales. There exists a strong rainfall gradient in the Williams river catchment. In the upper reaches, where elevations rise above 1500 m, the average rainfall is above 1600 mm. In the centre of the catchment the rainfall is at a minimum—the annual average for Dungog is under 1000 mm. Proximity to the coast influences rainfall in the south of catchment, with annual averages marginally above 1000 mm. Unfortunately, no long-term rainfall data is available in the upper reaches of the catchment. Hence, the three sites chosen do not represent the entire rainfall regime of the Williams river catchment. However, they will provide an indication whether a two-state long-term persistence structure exists in the rainfall regime of this important water supply catchment.

A long-term rainfall data set from Clarence Town Post Office (PO) which is located in the lower region of the Williams River catchment (Fig. 2(b)), was one of the high quality rainfall data sites produced by Lavery et al. (1997). Long-term rainfall records were also available at Dungog PO and Raymond Terrace. Dungog is centrally located within the Williams River catchment, and Raymond Terrace is located just south of the water supply catchment. Rainfall data from Dungog and Raymond Terrace were checked for inconsistencies by comparison with the high quality rainfall data from Clarence Town. There was no significant evidence of any inhomogeneities. A summary of the details of these three rainfall time series from the Williams river catchment is given in Table 1.

Fig. 3 shows the observed annual time series for all of the sites from the three regions used in this analysis. The annual data are aggregated to the May to April water year for the Warragamba and Central Coast sites and the April to March water year for the Williams river catchment data (Sections 5.2–5.4, outline the reasons for selecting these water years). These plots provide a preliminary indication of the long-term wet and dry periods in the observed data, particularly for sites such as Cataract Dam where the lower rainfall in the first half of this century is clearly evident. One possible explanation for these distinct shifts in the rainfall regime is that the series are nonstationary—due to other changes in measurement/observation technique or shifts in the climatic regime. As the long-term wet and dry periods are evident in the Cataract Dam rainfall data which was part of the high quality data set checked for changes in measurement/observation technique by Lavery et al. (1997) this reason can almost certainly be ruled out. This leaves shifts in the climatic regime as the most likely cause of the long-term wet and dry periods. By applying a stationary stochastic model such as the HMM to simulate these long-term wet and dry periods it is assumed that these changes in the climate regime are reversible and will occur again in the future.

### 5. Analysis of calibration results

A multi-site analysis was undertaken for the rainfall data from the Warragamba, Central Coast and the Williams River catchment regions. For the Warragamba catchment rainfall data only the rainfall sites that are located inside the catchment were included in the first analysis, then in a second analysis all the sites from the region were included. This will illustrate the influence of conducting analyses with a different number of sites. For the Central Coast and Williams river catchment rainfall data only one multi-site analysis was undertaken with all the available data from the region.

#### 5.1. Methodology

The objective of this multi-site analysis is to determine whether the assumptions of the HMM framework (outlined in Section 2) are justified and to determine if a strong persistence structure is evident in the rainfall regime of the Warragamba, Central Coast or Williams river catchment regions. To achieve this the posterior distribution $p(\theta|Y_{N}^{\text{obs}})$ of the relevant model parameters is examined. This allows direct assessment of the parameter uncertainty.
The methodology used to verify if each of these modelling assumptions is justified will now be outlined.

The first assumption of the multi-site HMM is that the distribution of rainfall at multiple sites is a mixture of two independent wet and dry state distributions. If the difference between these two distributions is not significant there is no justification for using a two-state model. In this study the wet and dry separation index (WADSI) is introduced as a measure of the difference between the wet and dry state rainfall distributions. The WADSI is defined as follows:

\[
\text{WADSI} = \frac{\mu_W - \mu_D}{\sqrt{\sigma_W^2 + \sigma_D^2}}
\]

Utilizing the assumption that both the state rainfall distributions are Gaussian it can be shown that the
probability of wet state rainfall being less than dry state rainfall \( P(y_W < y_D) \) is purely dependent on the WADSI statistic (Thyer, 2001). As the probability \( P(y_W < y_D) \) increases the difference between the wet and dry state distributions becomes more separated, i.e. when WADSI approaches zero the wet and dry state distributions become virtually indistinguishable, i.e. \( P(y_W < y_D) = 50\% \). As the WADSI increases the two distributions become more separated, i.e. when \( \text{WADSI} = 1.0 \) then \( P(y_W < y_D) = 16\% \) which implies the wet and dry states would be relatively easy to identify. The WADSI provides a convenient dimensionless measure of the wet and dry state separation which encapsulates the two important properties—the difference between the wet and dry state means and the influence of the wet and dry state standard deviations. The posterior of the WADSI can be calculated from the posterior samples for the wet and dry state rainfall parameters. If there is a high posterior density for WADSI values close to zero this would indicate that the wet and dry difference is insufficient to clearly identify each distribution. This would place doubt on the justification for the first assumption. Alternatively if there is a very low posterior density for WADSI values close to zero this would provide strong evidence to support this assumption.

The second assumption is that both the wet and dry state distributions are multi-variate Gaussian distributions. This assumption is tested by simulating rainfall values from the posterior predictive distribution, which is defined as the distribution of rainfall values from the posterior predictive distribution, the following methodology was used: (1) draw a sample \( \theta_s \) from \( p(\theta|y^\text{obs}) \). (2) Sample \( y^\text{rep} \) of length \( N \) from \( p(y|\theta_s) \). In this analysis 10,000 samples from \( p(\theta|y^\text{obs}) \) were generated by the Gibbs sampler, therefore 10,000 replicates of \( y^\text{rep} \) were generated. Using these 10,000 replicates the 5 and 95% probability limits of the sampling distribution of drawing \( N \) samples of \( y^\text{rep} \) were determined and compared to the observed data. If the observed data are within these 5 and 95% limits of \( p(y^\text{rep}|y^\text{obs}) \) the model is considered to be consistent with the data and hence there would be no reason to reject the second assumption. Inspection of \( p(y^\text{rep}|y^\text{obs}) \) also determines whether a significant proportion of the distribution is truncated because hydrological values cannot be negative.

The third assumption is that the current climate state, \( s_t \), is dependent on the previous climate state, \( s_{t-1} \). If the current climate state was independent of the previous climate state there would be no persistence in either the wet or dry state. For this case of state independence the transition probability posteriors would indicate the parameters are unidentifiable with values evenly distributed over the range 0–1. Therefore to test this assumption the transition probability posteriors are examined. The transition probability posteriors also provide an indication of the strength of the persistence structure. The expected state residence time, \( E(SRT) \), is simply the inverse of the transition probability value. On the diagrams displaying the transition probability posteriors the axes have been transformed to show the \( E(SRT) \), which has a range \( \infty \) to 1. These diagrams are shown in the form of posterior probability density contour plots. A matrix smoothing algorithm was applied to remove sampling noise. An artefact of this technique is that in some cases the posteriors extend beyond the limits of \( \infty \) to 1. This can be ignored; the overall trend is the important feature to note.

The fourth assumption of the HMM, that the state transition probabilities are stationary over time, was not tested because there is insufficient data to perform a meaningful split-sample test.

As yet there is no methodology to test the fifth assumption that every site has the same climate state at every point in time. The implications of this fifth assumption will be discussed in Section 6.

In all the analyses the monthly time series were aggregated to produce 12 annual vector time series each corresponding to the 12 ‘water’ years (January to December, February to January). Each of these 12 annual time series was used to calibrate the multi-site HMM. To measure which water year exhibits the strongest wet and dry state signal the state signal index (SSI), developed by Thyer and Kuczera (2000), was used. If a large proportion of the rainfall time series is equally likely to be classified as wet or dry, the states are considered to be poorly identified and SSI → 0. Conversely, if a high proportion of the rainfall time series is
consistently classified as either wet or dry, then the states are better identified and \( \text{SSI} \rightarrow 0.5 \).

A time series of the posterior probability that a particular year is from the wet state, \( P(s_t = W | Y_N) \), provides an indication of the wetter and drier periods in the historical time series. Referred to as the posterior of the hidden state time series, \( P(S_N | Y_N) \) will be examined for each of the multi-site analyses.

Thyer and Kuczera (2003) recommended that the priors should be compared to the posteriors. For all the calibration results presented in this study the priors were found to be relatively diffuse compared to the posteriors. In Section 5.2.1, an example of the least diffuse prior for all the results is given. For all the analyses the prior degrees of freedom \( n_0 \) was set to the minimum value of \( n_0 = r + 2 \), as recommended by Thyer and Kuczera (2003).

### 5.2. Warragamba catchment region

#### 5.2.1. Four-site analysis

The Mt. Victoria composite, Moss Vale, Taralga and Yarra composite rainfall time series were included in the four-site analysis. The January to December water year was found to have the highest SSI = 0.42. However inspection of the WADSI posterior for each site for each of the water years revealed that the January to December water year did not have the most number of sites with lowest probability density at WADSI = 0.0. This was the May to April water year. As the May to April water year also had a relatively high SSI = 0.35 it was chosen for further analysis. The WADSI posterior for each of the four sites is shown in Fig. 4(a). For all the sites except for Moss Vale there is a very low (practically zero) probability density at WADSI = 0.0. For Moss Vale although the probability density at WADSI = 0.0 is nonzero it is still reasonably low compared to the density at the mode. Overall, this is considered to indicate that it is highly likely that the first assumption of the multi-site HMM framework can be justified.

Fig. 5 shows that the observed data for the Mt. Victoria composite data set is within the 5 and 95% probability limits of the sampling distribution of the posterior predictive distribution. The proportion of this distribution that is assigned negative values is negligible (<0.1%). These results support the second assumption. Although not shown, similar results were found for all the other sites in all the multi-site analyses presented in this paper.

Fig. 4(b) shows that the transition probabilities are very well identified for this four-site analysis, which provides strong evidence to justify the third assumption. The \( P(S_N | Y_N) \) in Fig. 4(c) shows the well identified wet and dry periods. Table 2 gives the expected values from the posteriors for the four-site analysis.

Fig. 6 compares the prior to the posteriors for \( \sigma_w \) and \( \sigma_D \) of the Yarra composite site from this four-site analysis. For all the analyses considered in this study this site represents the worst case (i.e. least diffuse prior). Given that this worst case prior is acceptably diffuse over the range of the posterior, we therefore argue all the priors used in this study are adequately diffuse when compared to the posteriors.

#### 5.2.2. Five-site analysis

In the five-site analysis the Cataract Dam data was included. It was found that there was little difference in the SSI values between all the water years; they were all relatively high (>0.4). For comparison purposes the same May to April water year was used as the four-site analysis. This water year had a very high SSI = 0.43. The WADSI posterior for each of the five-sites is shown in Fig. 7(a). Again, for all the sites except for Moss Vale there is a very low probability density at WADSI = 0.0. For Moss Vale, the probability density at WADSI = 0.0 has increased compared to the four-site analysis. It is now quite close to the density at the mode. In a single site context the Moss Vale result would be classed as inconclusive. However, given the results for the other four sites the overall conclusion is that it is likely that a two-state persistence structure does exist in the Warragamba catchment rainfall regime. The implications of these Moss Vale results will be further discussed in Section 6.

Fig. 7(b) shows that the uncertainty of the transition probabilities for the five-site analysis significantly decreased compared to the four-site analysis (Fig. 4(b)). The \( P(S_N | Y_N) \) in Fig. 7(c) shows the long wet and dry periods. Table 2 gives
the expected values from the posteriors for the five-site analysis.

5.3. Central Coast catchment region—two-site analysis

In the two-site analysis for the Central Coast catchment region the Gosford and Wyee rainfall time series were included. The May to April water year had the highest SSI = 0.40, and also the lowest probability density at WADSI = 0.0 for both sites. Fig. 8(a) shows there is a very low probability density at WADSI = 0.0 for both sites. This provides significant evidence that it is highly likely the first assumption of the multi-site HMM is justified. Fig. 8(b) shows that the transition probabilities are well identified which provides strong evidence to justify the third assumption. The $p(S_N | Y_N)$ in Fig. 8(c)

Fig. 4. Posteriors for the four-site analysis of the Warragamba catchment rainfall data (May to April water year). For the transition probabilities the axes have been transformed to display the expected state residence time. The contour lines (from outside to in) correspond to the 99, 95, 90, 75, and 50% probability density regions.

Fig. 5. Comparison of the sampling distribution of drawing $N$ samples from the posterior predictive distribution $p(y_{rep} | Y_N)$ to the distribution of the observed data for the Mt. Victoria composite time series (May to April water year) from the four-site analysis of the Warragamba catchment rainfall data.
shows the well identified wet and dry periods. Table 2 gives the expected values from the posteriors for the two-site analysis.

### Williams River catchment region—three-site analysis

For the Williams River catchment rainfall data a three-site analysis was undertaken using Clarence Town, Dungog and Raymond Terrace. For the first analysis undertaken with the January to December water year one of the convergence diagnostics, the $R$ statistic, indicated that the Gibbs sampler did not achieve convergence. Inspection of the parameter samples for each of the 10 chains revealed that there seemed to exist two different modes in the posterior. The two different modes had hidden state time series which were the basically the inverse of one another, as shown in Fig. 9. One mode (denoted as mode 1) corresponded to a low separation in the wet and dry means for Clarence Town and a high separation for Dungog, while the other mode (denoted as mode 2) had the opposite, high wet and dry mean separation for Clarence Town and low separation for Dungog. For Raymond Terrace the separation of the wet and dry mean did change slightly at the two different modes but it was not as distinct as the other two sites. The state standard deviations for Dungog would also have the same separation as the modes as seen in Table 2.

#### Table 2

<table>
<thead>
<tr>
<th>Data set</th>
<th>Four-site analysis</th>
<th>Five-site analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u_W(\sigma_W)$ (mm)</td>
<td>$u_D(\sigma_D)$ (mm)</td>
</tr>
<tr>
<td>Mt. Victoria</td>
<td>1246 (319)</td>
<td>941 (251)</td>
</tr>
<tr>
<td>Moss Vale</td>
<td>1112 (288)</td>
<td>933 (240)</td>
</tr>
<tr>
<td>Taralga</td>
<td>927 (214)</td>
<td>743 (179)</td>
</tr>
<tr>
<td>Yarra</td>
<td>798 (185)</td>
<td>593 (146)</td>
</tr>
<tr>
<td>Cataract Dam</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$E(SRT)$ (years)</td>
<td>5.3</td>
<td>7.7</td>
</tr>
</tbody>
</table>

(a) Multi-site analysis of the Warragamba catchment rainfall data (May to April water year)

(b) Two-site analysis of the Central Coast region rainfall data (May to April water year)

(c) Three-site analysis of the Williams river catchment rainfall data (April to March water year)

#### Fig. 6

Comparison of the prior to the posterior for $\sigma_W$ and $\sigma_D$ for the Yarra composite site from the four-site analysis of the Warragamba catchment region rainfall data.
flip from $s_W$ being higher than $s_D$ for mode 1 to $s_D$ being higher than $s_W$ for mode 2. There was little change in remaining parameters for the different modes.

The reason that the $R$ statistic indicated convergence was not achieved was because some of the chains would only sample from one of these modes, and therefore the chains were not properly mixing in the parameter space. To alleviate this the number of samples for each of the 10 chains was increased from 1000 to 10,000 per chain to give a total of 100,000 samples from the posterior. The warm-up was also increased to 100,000 samples per chain. By increasing the number of samples it provided each of the chains the opportunity to visit both modes. Thus the chains were properly mixing and the $R$ statistic indicated convergence was achieved for the January to December water year. The bimodal posteriors are illustrated in the WADSI posterior shown in Fig. 10.

The two different modes at WADSI = 0.6 and WADSI = 0.0 can be seen in the Dungog and to a lesser extent the Clarence Town WADSI posterior. This diagram indicates that mode 1 was the major mode and mode 2 was only a minor mode.

When this method of increasing the number of samples was applied to the first four water years (January to December through to April to March) they all achieved convergence. For the April to March water year there was little evidence of the bimodal posterior with mode 1 being the dominant mode. This water year had the highest $SSI = 0.41$. Accordingly the results are presented for this water year. The WADSI posterior for each of the three sites is shown in Fig. 11(a). For Dungog and Raymond Terrace there is a very low (practically zero) probability density at WADSI = 0.0, indicating it is likely there are two distinct wet and dry distributions. For Clarence Town the mode of the posterior is located at WADSI = 0.0,

![Graphs showing posterior densities and transition probabilities](image)

Fig. 7. Posteriors for the five-site analysis of the Warragamba catchment rainfall data (May to April water year). Refer to Fig. 4 for explanation of transition probabilities diagram.
indicating there was no significant difference between the wet and dry rainfall distributions for this site. Fig. 11(b) shows that the transition probabilities are very well identified. This provides strong evidence to justify the third assumption. The $p(S_N|Y_N)$ in Fig. 11(c) shows the long wet and dry periods, although there is a different pattern to the Warragamba results (Figs. 4(c) and 7(c)). Table 2 gives the expected values from the posteriors.

For the remaining 8 water years (May to April through to December to November) convergence was not achieved. Again, the reason was because all the chains were not visiting both the modes in the posterior and therefore not mixing properly. Inspection of the iterative sequence of the parameter samples indicated that for these water years the proportion of the posterior attributed to mode 2 seemed to increase and the chains were not able to move as easily between modes 1 and 2, even with the increased samples. It is not possible to determine the proportion of the posterior attributed to modes 1 and 2 because convergence was not achieved, hence the samples do not accurately represent the posterior. Given that convergence was not achieved for all the water years the results presented for the Williams river catchment region can only be treated as preliminary. The reason for the existence of these two modes in the posterior is unknown at this stage. The implications of this finding will be further discussed in Section 6.

Fig. 8. Posteriors for the two-site analysis of the Central Coast catchment rainfall data. (May to April water year). Refer to Fig. 4 for explanation of transition probabilities diagram.

Fig. 9. Comparison of the wet state frequency from the two different modes in the posterior of the three-site analysis for the William River catchment region rainfall data (January to December water year).
6. Discussion

6.1. Selection of appropriate water year

The methodology developed for the single site HMM (Thyer and Kuczera, 2000) to evaluate which water year was chosen produced inconsistent results in the multi-site context for the Warragamba catchment region rainfall data. Unlike the single site results the water year with the highest SSI value did not have largest wet and dry difference for all sites. These results suggest that the methodology for choosing the appropriate water year is not suitable for the multi-site HMM. While the SSI value does give a good indication of the wet and dry state signal it should not be exclusively used for selecting the appropriate water year in the multi-site context.

For the rainfall data from both the Warragamba and Central Coast catchments the May to April water year was selected as the appropriate water year. This choice is also an intuitive one considering the climatology of the region. Both these catchments are located in a region where summer weather systems generally stem from the tropical Pacific Ocean, whereas winter systems originate from the Southern Ocean cold fronts. The autumn period (March to May) is typically the time of transition between the two weather patterns. In addition, the El Niño-Southern Oscillation (ENSO) cycle plays a significant role in the rainfall patterns of these regions.

![Fig. 10. Posterior of the WADSI for the three-site analyses of the Williams river catchment region rainfall data for the January to December water year.](image)

**Fig. 10.** Posterior of the WADSI for the three-site analyses of the Williams river catchment region rainfall data for the January to December water year.

![Fig. 11. Posterior for the three site analysis of the Williams River catchment rainfall data (April to March water year).](image)

**Fig. 11.** Posterior for the three site analysis of the Williams River catchment rainfall data (April to March water year). Refer to Fig. 4 for explanation of transition probabilities diagram.

![Fig. 12. Expected dry state residence time and expected wet state residence time for the Warragamba catchment region rainfall data.](image)

**Fig. 12.** Expected dry state residence time and expected wet state residence time for the Warragamba catchment region rainfall data.
Nin˜o phenomenon is known to be phase locked to the annual cycle (Allan et al., 1996; Nicholls, 1992). The wet periods associated with a La Niña and the dry periods associated with an El Niño generally start and finish in the autumnal period. This corresponds to the choice of the May to April water year. However, the state persistence identified in Figs. 4(c), 7(c), 8(c), and 11(c) indicates a cycle longer than the duration of a typical ENSO pattern (1–2 years).

6.2. Simulation of autocorrelation function

The autocorrelation function (ACF) is often used as a measure the long-term persistence for a given time series. Therefore, it is important to test the ability of the HMM to reproduce the ACF of the observed time series. This was done by calculating the ACF for each of the 10,000 replicates of simulated time series \( \gamma^{(p)} \) (refer Section 5.1) and then deriving the 90% confidence limits for the simulated ACF. Fig. 12 compares the observed and simulated ACFs for the Mt. Victoria site from the five-site analysis of the Warragamba catchment. It can be seen that at certain lags the observed values are outside the simulated 90% confidence limits. It is possible to test whether the number of observed autocorrelations outside the 90% limits is statistically significant assuming a null hypothesis that the HMM adequately simulates the observed ACF. If \( X \) is the number of lags outside the 90% confidence limits and \( X_{\text{obs}} \) is the observed number, then assuming independence the binomial distribution can be used to calculate

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P(X \geq X_{\text{obs}})\, \text{for the 30 lags used in the ACF. If this probability is lower than a given significance level then there is sufficient evidence to reject the null hypothesis. Table 3 shows that for all the sites from the four- and five-site Warragamba catchment analysis and the two-site Central Coast analysis there is insufficient evidence to reject the null hypothesis at the 5% significance level. Because the sites are spatially correlated the test statistics for each site are not independent. Nonetheless, the prima facie evidence is that HMM produces ACFs consistent with the observed data.}

6.3. Influence of multi-site data—comparison to single site results

One of the advantages of the multi-site HMM given in the introduction was that space-for-time substitution could potentially enhance the likelihood of identifying the two-state persistence structure. Thyer and Kuczera (2003) found using synthetic multi-site data that the identification of the persistence structure was improved when the correlation between the sites was not high. This was for the ideal case when the sites had the same hidden state time series and the same wet and dry state distributions. For real data it is unknown whether sites have the same hidden state time series and it is unknown whether the wet and dry difference is the same at different sites. The influence of multi-site data on the identification of a two-state persistence structure is investigated by comparing the posteriors of the transition probabilities from the single site to the multi-site analyses. If the uncertainty in the transition probabilities decreased this is interpreted as an improvement.

![Fig. 12. Comparison of observed and simulated ACFs for the Mt. Victoria site from the Warragamba catchment five site analysis.](image-url)
in the identification of the two-state persistence structure. Thyer (2001) presented the single site HMM results for the Warragamba catchment and Williams River catchment rainfall data used in this study. For the Warragamba data these single site results are compared to the multi-site results in Fig. 13. It should be noted that the Mt. Victoria and Taralga single site results had quite different water years (January to December) to the multi-site results. Whereas for the other sites (Moss Vale, Yarra and Cataract Dam) the water years were similar to the multi-site results. In the single site results Moss Vale and Taralga had the worst identified two-state persistence structure, which improved considerably when sites which had better identified persistence structures were included (Mt. Victoria and Yarra) in the four-site analysis. Conversely, the well-identified persistence structure of Mt. Victoria worsened when Moss Vale and Taralga were included. The effect of adding a site (Cataract Dam) with a large wet and dry separation and well identified persistence structure is clearly seen. The two-state persistence structure becomes very well identified and its strength increased considerably. These results indicate that if a site has a well identified persistence structure and/or a large wet and dry separation then it is likely to enhance the identification of the persistence structure, whereas if it does not then it is unlikely to improve the identification of the persistence structure.

For the Central Coast catchment region single and two-site results for the same water year are compared in Fig. 14. For Wyee the identification of the persistence structure improved considerably from the single to the two-site analysis. Gosford also showed some improvement in the identification of the persistence in the two-site analysis even though the Wyee single site results did not have a very well identified persistence structure. This indicates that the power of multi-site data can improve the identification of the persistence structure.

![Fig. 13. Comparison of the posteriors for the transition probabilities for the single site, four-site and five-site analysis for the Warragamba catchment rainfall data (May to April water year).](image1)

![Fig. 14. Comparison of the posteriors for the transition probabilities for the single site and two-site analysis for the Central Coast catchment region rainfall data (May to April water year).](image2)
The Williams River data results are compared in Fig. 15. The single site results indicated that the two-state persistence structure was not very well identified for all three sites (Thyer, 2001). However, when all the data was combined in a three-site analysis the identification of the persistence structure improved markedly. This result was pleasing because it was somewhat surprising that a two-state persistence structure was not well-identified in the single site Williams River results given that it is located only approx. 200 km north of the Warragamba catchment. It is noted that the Williams river catchment rainfall data did have a shorter record than the Warragamba rainfall data, which is likely to be a contributing factor. Again, this suggests that the power of multi-site data can improve the identification of the persistence structure. Unfortunately, this conclusion is tempered by the results that the wet and dry separation for Clarence Town indicated it was likely there was little wet and dry separation. Given that Clarence Town is located between Dungog and Raymond Terrace (refer Fig. 2(b)) this result is a concern. Also because convergence could not be achieved for the majority of water years in the three-site analysis these results must be treated as preliminary.

Overall, these results do suggest that space-for-time substitution when using multi-site data can improve the identification of the two-state persistence if it was not very well identified in a single site analysis. Given the results from the synthetic calibration runs (Thyer and Kuczera, 2003) the authors are hesitant to suggest that this conclusion can be applied generally. As part of future research it recommended that more synthetic calibration runs be undertaken to determine what factors affect whether multi-site data enhances the identification of the two-state persistence structure.

6.4. Identification of a homogenous persistence region

The fifth assumption of the multi-site HMM is that every site is in the same climate state at every point in time. This implies that all the sites included in the analysis are located within a homogenous persistence region. Initially, several techniques which were based on comparing the $p(S_0|Y_N)$ for each site derived from fitting the single site HMM to each site individually were trialled to provide a preliminary indication whether sites were from the same homogeneous region. However, no suitable methodology that was consistent with the multi-site results was found. Therefore, a robust methodology for deciding whether a site belongs to a certain homogenous persistence region has yet to be developed. The results for the four- and five-site analysis for the Warragamba catchment give the impression that the Moss Vale data may not be a part of the same persistence region as the other sites. It is possible that Moss Vale is from the same persistence region except that the wet and dry difference is not as clearly defined as for the other sites, or alternatively, Moss Vale may have a slightly different persistence structure that is swamped by the weight of rainfall information from the other three sites. When the Cataract Dam data is included in the five-site analysis the wet and dry difference
significantly decreases for Moss Vale, and also slightly decreases for the other three sites—compare Figs. 4(a) and 7(c). For Moss Vale this decrease is such that there is considerable doubt whether the first HMM assumption is justified. However, there is a large increase in the strength of the identified persistence structure for the five-site analysis. If Moss Vale is removed and the remaining four-sites analysed the uncertainty in the transition probabilities increases slightly, indicating the persistence structure is not as well identified. This would not happen if Moss Vale was not part of the same persistence region as the other sites. Overall, these results present no clear indication whether Moss Vale is part of the same persistence region as the other four sites.

The $p(S_{SN}|Y_N)$ for the two-site analysis for the Central Coast region given in Fig. 8(a) is very similar to the results for Sydney given in Thyer and Kuczera (2000). This provides strong evidence to suggest a homogeneous persistence exists in the region identified by these three sites (Fig. 2(a)). Comparison to the $p(S_{NL}|Y_N)$ results for the Warragamba catchment region (Figs. 4(c) and 8(c)), which is also located close to Sydney, show that the dry period in the first half of this century is common, however, the second half has quite different wet and dry periods.

One of the major assumptions of the multi-site HMM is that each site has a common persistence structure. Now, although there is strong evidence that the four sites from the Warragamba catchment region and the sites from the Central Coast catchment region have a common regional persistence structure during certain time periods the Moss Vale results highlight the need for the development of methodology to decide whether a site belongs to a certain homogenous persistence region. This methodology will likely be based on a statistical measure of the goodness of fit of the multi-site HMM which assumes a common persistence structure compared to the single site HMM which assumes an independent persistence structure for each site. In the research applying the nonhomogenous HMM, Hughes et al. (1999) used the Bayesian information criteria (BIC) to decide which climate predictors and model structures provided the most parsimonious model that fits the data well. Gelman et al. (1995) mention that Bayes factors can be used to compare competing models to decide which provides the better fit to the data. This suggests that the application of the BIC or Bayes factors to the multi-site HMM is worthy of future investigation as a method for identifying which sites belong to a certain homogenous persistence region.

6.5. Influence of sampling for missing data

The motivation for developing a methodology for handling missing data was to ensure that all the available rainfall information would be utilised. Otherwise in a multi-site analysis all the time series would have to be truncated to the length of the shortest time series, thereby losing potentially valuable rainfall information. To ascertain if there is any advantage gained by sampling missing data the multi-site analyses of the Warragamba and Central Coast catchments regions were repeated with the time series truncated to the shortest time series (i.e. no sampling of missing data). If the identification of the long-term persistence structure worsens with these reduced data sets then this indicates there is an advantage to sampling for missing data. Similar to Section 6.4 a change in the uncertainty of the transition probabilities is used to indicate a change in the identification of the long-term persistence structure. For the Warragamba data an analysis without sampling for missing data means the time series is reduced from 123 to 112 years for the four-site analysis and from 123 to 94 years for the five-site analysis. The resulting transition probability posteriors are compared to their original posteriors in Fig. 16 for the Warragamba catchment region. It can be seen there was only a very slight change in the posteriors with or without sampling missing data. Similarly, the WADSI posteriors also showed little difference. For the Central Coast data the time series was reduced from 123 to 93 years when missing data is not sampled. Fig. 17 shows there was a bigger increase in the uncertainty of the transition probabilities when the missing data is not sampled than the Warragamba data. Also, the WADSI posterior indicated the wet and dry difference for Wyee was not as strong when the time series was truncated. This difference in the Central Coast and Warragamba results is probably due to the greater number of sites in the Warragamba analysis ensuring the long-term persistence structure remains well-identified even when the time series is truncated slightly.
The results for the Central coast catchment illustrate the utility of having a methodology to sample for missing data to ensure all the rainfall information is utilised. In other applications when the multi-site data are not continuous at one or more sites it would also be advantageous to sample the missing data. However, it must be remembered every year of missing data is an extra parameter that must be identified. Therefore, in cases where there is a large amount of missing data (e.g. one site with 30 years and another with 100 years of observed data) it is anticipated that sampling for missing data (70 extra parameters for the example given) would not improve the identification of the long-term persistence structure. From a practical point of view this is a significant limitation of the current approach for handling missing data. Future research will investigate alternative approaches that overcome this limitation (Thyer and Kuczera, 2003).

6.6. Implications of calibration results

The results for both the Warragamba and Central Coast catchments provided strong evidence that the data favoured a stationary model with two-state persistence over a single-state stationary model with no persistence. Of course there maybe other conceptualisations of long-term persistence which are supported by the data. However, it is not an objective of this study to discriminate between such models.

If the current climate state is known it is possible to forecast future annual rainfall distributions using a similar procedure to that outlined for the single site HMM (Thyer and Kuczera, 2000). This is useful for
investigating how long the predicted distribution is drier or wetter than the long-term observed distribution when the climate state has been dry or wet, respectively. This method is used to simulate the predicted distribution of rainfall for the Yarra composite site at time intervals of 1, 2, 5 and 10 years into the future. Two different cases are considered, the first case assumes the current year’s climate state is dry and the second assumes it is wet.

Fig. 18 shows the cumulative distribution of the observed Yarra composite data compared to the predicted distributions conditioned on wet and dry initial states. For the case when the current year is dry the predicted distribution of rainfall 1 and 2 years into future is distinctly lower than the observed distribution (Fig. 18(a)). It is not until 5 years time that the predicted distribution becomes similar to the observed distribution. The magnitude of the difference is greater for the case when the current year is wet (Fig. 18(b)). It is worth noting that unlike the current year dry case there is still a slight difference between the predicted rainfall at 5 years into the future and the observed distribution. The magnitude of the difference is greater for the case when the current year is wet (Fig. 18(b)). It is worth noting that unlike the current year dry case there is still a slight difference between the predicted rainfall at 5 years into the future and the observed distribution. Initially, this result seemed counterintuitive because the dry state persistence is stronger than the wet state persistence. However, upon comparing the posterior expected values for the wet and dry mean (Table 2) to the mean of the entire series (Table 1) it can be seen that the dry state distribution is closer to the observed marginal distribution than the wet state distribution.

Figs. 4(c) and 7(c) show for the Warragamba catchment, which is Sydney’s most important water supply catchment, the first half of this century was predominantly dry compared to the second half of this century. What are the implications for water resource management for this region if such a dry period were to occur again? To provide a preliminary indication of the likelihood of these longer duration dry periods the posterior predictive distribution was used to generate the probability distribution of the longest wet and dry state spells in 123 years. Multi-site rainfall data of 10,000 simulations were generated by the multi-site HMM using the posteriors from the four-site analysis. The results are shown in Fig. 19. The probability that the longest spell in the dry state will be greater than 50 years is approx. 10%. This is a reasonably high probability given that the expected dry state residence time is only approx. 8 years. This illustrates that
despite its purely Markovian structure the HMM has the ability to produce spells in the dry state with a very long duration. However, it is worth making a comment on the interpretation of these results. A spell in a dry state does not necessarily correspond to ‘drought’ conditions. It means that there is an increased chance of lower rainfall. The difference between the dry state mean and the empirical mean for all the sites in the Warragamba four-site analysis is in the range 7–13%, which is not large. Nonetheless, given the nonlinearity in the Australian rainfall-runoff transformation, it is likely that such a difference will be enhanced in the streamflow time series.

6.7. Alternative multi-site HMM frameworks

The results for the Williams river catchment were included to illustrate that the intricacies of the multi-site HMM and its calibration procedure are not fully understood. It is unknown why the posteriors for the Williams exhibit bimodal behaviour. It is possible that one contributing factor may be that the sites are highly correlated (close to 0.9). Thyer and Kuczera (2003) found using synthetic data that when sites have a high correlation structure the posterior variance is likely to increase. In the future it is recommended that further synthetic calibrations be undertaken to develop an interpretative framework for the multi-site HMM results. To have an understanding of what factors affect the identification of the regional two-state persistence would be very valuable.

Synthetic calibration runs led to the development of a more suitable prior specification (Thyer and Kuczera, 2003). Similar insight and refinements may be gained by undertaking more synthetic calibration runs.

The evidence of bimodal posteriors in the Williams River catchment region rainfall data, especially the flipping of the hidden state frequency time series between the two modes, suggests that the current HMM structure may not suitable for this region. Potentially, a three-state HMM, with a normal, wet and a dry state may be more appropriate. If the rainfall data did have three states, then the two modes could be interpreted as the model trying to force the wet and normal state rainfall into one state or forcing the dry and normal rainfall into the other state. Given that Franks (1998) and Wooldridge et al. (2002) illustrated the influence of El Niño in the Williams river catchment by partitioning hydrological data into a dry (El Niño) phase, a normal phase, and a wet (La Niña) phase a three-state HMM maybe more appealing in a climatological sense.

Another alternative multi-site HSM framework, which could alleviate the convergence problems that occurred with the Williams River catchment rainfall data, is to assume that the wet and dry states have the same spatial correlation matrix. The posteriors for the wet and dry state covariance matrix provide no significant evidence that such an assumption would not be valid. The advantage of this approach is that it would reduce the parameterisation of the multi-site HMM. For example, in a four-site analysis the number of parameters is reduced from 30 to 24. Such a simplification may eliminate convergence problems, especially with highly correlated sites where little extra rainfall information is provided by the multi-site data.

The difference in the results between the Warragamba and Williams River catchments leads us to question the assumption that there does exist a regional climate state. The Williams River catchment is located only approx. 200–250 km north of the Warragamba catchment. It would be expected that large-scale circulations such as El Niño would impact on regions much larger than the distance between these two catchments. Alternatively it may be that the assumption of a common climate state across all sites may be too strong—especially over large regions. At certain points in time a common climate state may exist, and other times there may be a high spatial variability in the climate state. Consider the 1982–1983 El Niño. It caused widespread droughts over a large portion of Australia, whereas other El Niño events caused lower rainfall to occur in only certain regions of Australia. The reason for this spatial variation in the influence of these climatic mechanisms is not well understood, but it is known that it does exist.

A possibly alternative multi-site HMM could allow for this spatial variation while still maintaining a link in the climate state between the sites. The extent of this spatial variation in the climate state could possibly be linked to climatic predictors following the procedure developed by Hughes et al. (1999). This could provide valuable insight for understanding
the physical mechanisms that produce long-term persistence. The multi-site HMM framework, in its current form, is very useful tool for drought risk assessment purposes where long-term simulations of hydro-climatic inputs are required. However, for water resource management on a year-to-year basis, it would be extremely useful to have an understanding of the climatological reasons for the occurrence of these long dry periods, such as the one which occurred in the first half of this century. A multi-site HMM framework that incorporated climatic information has the potential to provide such insight.

The WADSI is a convenient measure to determine if there is a difference between the expected rainfall in the wet and dry states. However, it is not able to detect ‘climate states’ where there is a difference in the standard deviations, but not the means. This unnecessarily restricts the interpretation of the climate states. For example, climate states could represent cases where the rainfall is ‘more variable’ and ‘less variable’ rather than simply ‘wet’ and ‘dry’. The multi-site HMM model can detect this type of persistence. For example, Table 2 shows that expected value from the posterior for $s_W$ was higher than $s_D$ for almost all the analyses undertaken in this study. This suggests that alternative measures should be developed to complement the WADSI to allow for a more flexible interpretation of the states rather than simply wet or dry.

7. Conclusions

A multi-site HMM was calibrated to long-term multi-site rainfall data from three important water supply catchments on the east coast of Australia. The use of a multi-site HMM to simulate long-term persistence in multi-site rainfall time series represents a new application of the HMM framework in stochastic hydrology. A Bayesian MCMC method known as the Gibbs sampler was used to infer the posterior distribution of the HMM parameters. Knowledge of the posteriors enabled methodologies to be developed to test the majority of the multi-site HMM assumptions.

For the rainfall data from the Warragamba and Central Coast catchment regions the results indicated that the multi site HMM with two-state persistence was more likely than a single state model with no persistence. The implications of this finding may have a considerable effect on water resource management and drought risk assessment for the regions supplied by these two major water supply sources. Comparison with the single-site results showed that the multi-site data enhanced the identification of the long-term persistence structure. The benefits of having a methodology of sampling for missing data to ensure all the rainfall information is utilized were also demonstrated.

For the rainfall data from the Williams River catchment region the existence of two modes in the posteriors meant it was difficult to achieve convergence of the Gibbs sampler. However, preliminary results suggest that a two-state persistence structure may also exist in this important water supply catchment. At this stage, it is unknown why these two modes exist and this illustrates that the subtleties of the multi-site HMM are not yet fully understood.

The development of the multi-site HMM and its calibration procedure represents a significant advance in the modelling of long-term persistence of hydrological time series at multiple sites. However, the results presented in this paper highlighted that there are still several crucial issues that require further investigation. These include developing a methodology for identifying a homogenous persistence region and using synthetic data to develop a more comprehensive understanding of the effect of multi-site data and the effect of sampling missing data on the identification of a two-state persistence structure. Also, it was concluded that alternative multi-site HMM frameworks such as a three-state model, or a framework that allowed for spatial variation in the climate state or the utilization of climatic predictors warrant attention.

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