Learning Simplicial Complexes from Persistence Diagrams

Robin Lynne Belton\textsuperscript{1}  
Brittany Terese Fasy\textsuperscript{1,2}  
Rostik Mertz\textsuperscript{2}  
Samuel Micka\textsuperscript{2}  
Anna Schenfisch\textsuperscript{1}  
David L. Millman\textsuperscript{2}  
Daniel Salinas\textsuperscript{2}  
Jordan Schupbach\textsuperscript{1}  
Lucia Williams\textsuperscript{2}  

08 August 2018, Winnipeg  
CCCG 2018

\textsuperscript{1}Depart. of Mathematical Sciences, Montana State U.  
\textsuperscript{2}School of Computing, Montana State U.
Collaborator Recognition
Motivation

- Topological descriptors can be used as a statistic for shape comparison and classification.
- New methods of storing and representing geometric objects.

Problem
Graphs

Graph \( G = \langle V, E \rangle \) where \( V \) are the vertices...
Graph $G = \langle V, E \rangle$ where $V$ are the vertices and $E$ are the edges.
Assumption: $G$ has a straight-line embedding in $\mathbb{R}^2$ and is planar.
Filtration Intuition

Filtration on graph $G$: sequence of subgraphs $G_0 \ldots G_n$ such that if $j \leq i$ then $G_j \subset G_i$. 
Filtration Intuition

Filtration on graph $G$: sequence of subgraphs $G_0 \ldots G_n$ such that if $j \leq i$ then $G_j \subset G_i$. 
Filtration Intuition

Filtration on graph $G$: sequence of subgraphs $G_0 \ldots G_n$ such that if $j \leq i$ then $G_j \subseteq G_i$. 
As subgraphs are discovered, new features (connected components) are born and killed.
Example with height filtration

As subgraphs are discovered, new features (connected components) are born and killed.
As subgraphs are discovered, new features (connected components) are born and killed.
Background

Example with height filtration

As subgraphs are discovered, new features (connected components) are born and killed.
Example with height filtration

As subgraphs are discovered, new features (connected components) are born and killed.
As subgraphs are discovered, new features (connected components) are born and killed.
Previous Work

• Turner et al. Show that persistence diagrams are representative of underlying three-dimensional shape $^a$.
• Require infinite number of directions to offer exact representation.
• We extend theoretically with algorithm for determining the number of directions necessary and how to choose them.

$^a$Turner, K., Mukherjee, S., and Boyer, D. M. Persistent homology transform for modeling shapes and surfaces. Information and Inference: A Journal of the IMA 3, 4 (2014), 310–344

Figure 13: Images of a calcaneus from two different angles.
Ultimate Goal

Learn Simplicial Complexes from Persistence Diagrams
Vertex Locations

General Idea
- Height index filtration on simplicial complexes
- Reconstruction done with three persistence diagrams
Vertex Locations
Vertex Locations

![Diagram of vertex locations](image)
Vertex Locations

![Diagram of vertex locations with labels and axes]

$s_1$
Vertex Locations
Vertex Locations
Vertex Locations

Vertex Locations
Vertex Locations

Theorem 5 (Vertex Reconstruction)
Vertex Locations

Theorem 5 (Vertex Reconstruction)
Vertex Locations

Theorem 5 (Vertex Reconstruction)
Determining Edge Locations

We have vertex locations...

How do we test if there exists an edge between two vertices?
Indegree of Vertex

Indegree of vertex

![Diagram of a vertex with indegree 4]
Indegree of Vertex

\[ \text{Indegree of vertex} \]

\[ \text{INDEG}(v, s) = |\{(x, y) \in D_0(s) \mid y = v \cdot s\}| + |\{(x, y) \in D_1(s) \mid x = v \cdot s\}|. \]

**Lemma 7 (Indegree from Diagram)
Indegree of Vertex

Indegree of vertex

\[ \text{Indeg}(v, s) = |\{(x, y) \in D_0(s) \mid y = v \cdot s\}| + |\{(x, y) \in D_1(s) \mid x = v \cdot s\}|. \]

Lemma 7 (Indegree from Diagram)
Indegree of Vertex

Indegree of vertex

\[ \text{INDEG}(v, s) = |\{(x, y) \in D_0(s) \mid y = v \cdot s\}| + |\{(x, y) \in D_1(s) \mid x = v \cdot s\}|. \]

Lemma 7 (Indegree from Diagram)
Determining Edge Locations
Determining Edge Locations

Lemma 9 (Edge Existence)
Edge Locations

Determining Edge Locations

Lemma 9 (Edge Existence)
Theorem 10 (Edge Reconstruction)
Conclusion

Overall Idea

Summary

Theorem 11 (Plane Graph Reconstruction)
Future Work

In Progress

- Remove reliance on diagonal.
- Utilize Euler Characteristic Curves.
- Implement to test effectiveness of classification.
- Extend to higher dimensions and different types of complexes.
- Explore new representations and storage approaches for simplicial complexes.
Questions?

Thanks!

\[ s_1 \rightarrow s_2 \rightarrow s_3 \]

samuel.a.micka@gmail.com