Challenges in Reconstructing Shapes from Euler Characteristic Curves

Brittany Terese Fasy\textsuperscript{1 2} Samuel Micka\textsuperscript{1} David L. Millman\textsuperscript{1}
Anna Schenfisch\textsuperscript{2} Lucia Williams\textsuperscript{1}

October 26, 2018
FWCG 2018 (Queens College)

\textsuperscript{1}School of Computing, Montana State U.
\textsuperscript{2}Depart. of Mathematical Sciences, Montana State U.
Collaborator Recognition

[Image of five profiles]
Outline

Talk Outline

- Motivation
- Background
- Previous Work and Problem Statement
- Previous Approach (Belton et al. (2018))
- Euler Characteristic Curves
- Summary and Future Work
Motivation

- Topological descriptors such as persistence diagrams (PDs) and the Euler characteristic can be used to describe shapes.
Motivation

- Topological descriptors such as persistence diagrams (PDs) and the Euler characteristic can be used to describe shapes.
- Specific collections of these topological descriptors can be used to represent geometry and embedding of shapes as well.
Motivation

- Topological descriptors such as persistence diagrams (PDs) and the Euler characteristic can be used to describe shapes.
- Specific collections of these topological descriptors can be used to represent geometry and embedding of shapes as well.
- These descriptors have applications in shape comparison.
Outline

Talk Outline

- Motivation
- **Background**
- Previous Work and Problem Statement
- Previous Approach (Belton et al. (2018))
- Euler Characteristic Curves
- Summary and Future Work
Graphs

Graph $G = \langle V, E \rangle$ where $V$ are the vertices...
Graph $G = \langle V, E \rangle$ where $V$ are the vertices and $E$ are the edges
Assumption: G has with straight-line embedding in $\mathbb{R}^2$ and is planar.
Euler Characteristic

- *Euler Characteristic* is a topological descriptor.
- Alternating sum of number of \( p \)-simplices, denoted \( \chi \).
  - e.g., \(|Vertices| − |Edges|\)

![Diagram of a graph with vertices and edges]
Euler Characteristic

- *Euler Characteristic* is a topological descriptor.
- Alternating sum of number of $p$-simplices, denoted $\chi$.
  - e.g., $|Vertices| - |Edges|$

\[ \chi = |V| - |E| = 9 - 9 = 0 \]
Background

Filtration Intuition

Filtration on graph $G$: sequence of subgraphs $G_0 \ldots G_n$ such that if $j \leq i$ then $G_j \subset G_i$. 
Filtration Intuition

Filtration on graph $G$: sequence of subgraphs $G_0 \ldots G_n$ such that if $j \leq i$ then $G_j \subset G_i$. 
Filtration Intuition

Filtration on graph $G$: sequence of subgraphs $G_0, \ldots, G_n$ such that if $j \leq i$ then $G_j \subset G_i$. 
As subgraphs are discovered, the Euler Characteristic changes. We refer to the function describing these changes as the *Euler Characteristic Curve* (ECC).
As subgraphs are discovered, the Euler Characteristic changes. We refer to the function describing these changes as the *Euler Characteristic Curve* (ECC).
As subgraphs are discovered, the Euler Characteristic changes. We refer to the function describing these changes as the *Euler Characteristic Curve* (ECC).
As subgraphs are discovered, the Euler Characteristic changes. We refer to the function describing these changes as the *Euler Characteristic Curve* (ECC).
As subgraphs are discovered, the Euler Characteristic changes. We refer to the function describing these changes as the *Euler Characteristic Curve* (ECC).
As subgraphs are discovered, the Euler Characteristic changes. We refer to the function describing these changes as the *Euler Characteristic Curve* (ECC).
As subgraphs are discovered, the Euler Characteristic changes. We refer to the function describing these changes as the *Euler Characteristic Curve* (ECC).
As subgraphs are discovered, the Euler Characteristic changes. We refer to the function describing these changes as the *Euler Characteristic Curve* (ECC).
As subgraphs are discovered, the Euler Characteristic changes. We refer to the function describing these changes as the Euler Characteristic Curve (ECC).
As subgraphs are discovered, the Euler Characteristic changes. We refer to the function describing these changes as the *Euler Characteristic Curve* (ECC).
Background

Outline

Talk Outline

- Motivation
- Background
- Previous Work and Problem Statement
  - Previous Approach (Belton et al. (2018))
  - Euler Characteristic Curves
- Summary and Future Work
Previous Work

Turner et al. (2014) show that infinite persistence diagrams (and ECCs) generated from lower-star filtration are representative of underlying two- and three-dimensional simplicial complexes. Proof method uses reconstruction of complexes from persistence diagrams.

Belton et al. (2018) show that a finite number of persistence diagrams can reconstruct plane graphs in $\mathbb{R}^2$.

Curry et al. (2018) show that reconstruction possible with finite topological descriptors and assumptions about curvature.
Problem Statement

Ultimate Goal

Learn Simplicial Complexes from ECCs

- There exists a linear number of Euler Characteristic Curves that are capable of determining vertex locations (similar observation made by Curry et al. (2018) in Lemma 7.2).
Ultimate Goal

Learn Simplicial Complexes from ECCs

- There exists a linear number of Euler Characteristic Curves that are capable of determining vertex locations (similar observation made by Curry et al. (2018) in Lemma 7.2).

- Can we reconstruct vertex locations of plane graphs using a finite number of ECCs, e.g. extend vertex results from Belton et al. (2018) using ECCs?
Outline

Talk Outline

- Motivation
- Background
- Previous Work and Problem Statement
- **Previous Approach (Belton et al. (2018))**
- Euler Characteristic Curves
- Summary and Future Work
Vertex Locations

First, we find the vertex locations.
**Vertex Locations**

**Vertex Locations using PDs (Belton et al. (2018))**

The $p$-th persistence diagram summarizes the “size” of topological features from the $p$th homology group.

Vertex reconstruction from Belton et al. (2018).
Vertex Locations

The $p$-th persistence diagram summarizes the “size” of topological features from the $p$th homology group.

Vertex reconstruction from Belton et al. (2018).
Vertex Locations

Vertex Locations using PDs (Belton et al. (2018))

The $p$-th persistence diagram summarizes the “size” of topological features from the $p$th homology group.

Vertex reconstruction from Belton et al. (2018).
### Vertex Locations

**Vertex Locations using PDs (Belton et al. (2018))**

The $p$-th persistence diagram summarizes the “size” of topological features from the $p$th homology group.

Vertex reconstruction from Belton et al. (2018).
Vertex Locations

Vertex Locations using PDs (Belton \textit{et al.} (2018))

The $p$-th persistence diagram summarizes the “size” of topological features from the $p$th homology group.

Vertex reconstruction from Belton \textit{et al.} (2018).
Vertex Locations

Vertex Locations using PDs (Belton et al. (2018))

Vertex reconstruction from Belton et al. (2018).
Vertex Locations

Vertex Locations using PDs (Belton et al. (2018))

Vertex reconstruction from Belton et al. (2018).
Vertex Locations

Vertex Locations using PDs (Belton et al. (2018))

Vertex reconstruction from Belton et al. (2018).
Vertex Locations

Vertex Locations using PDs (Belton et al. (2018))

Vertex reconstruction from Belton et al. (2018).
Talk Outline

- Motivation
- Background
- Previous Work and Problem Statement
- Previous Approach (Belton et al. (2018))
- Euler Characteristic Curves
- Summary and Future Work
Vertex Locations

Vertex Locations Using ECC

Vertex reconstruction using technique from Belton et al. (2018) with ECCs.
Vertex Locations Using ECC

Vertex reconstruction using technique from Belton et al. (2018) with ECCs.
Vertex Locations

Vertex Locations Using ECC

Vertex reconstruction using technique from Belton et al. (2018) with ECCs.
Vertex Locations

Vertex Locations Using ECC

Vertex reconstruction using technique from Belton et al. (2018) with ECCs.
Plane Graphs with ECCs

Degrees of different vertices

*Witness line* is a line associated with a change in the ECC and a vertex in the plane graph. When do ECCs yield witness lines?
**Plane Graphs with ECCs**

**Degrees of different vertices**

*Yes*

*Yes*

*Witness line* is a line associated with a change in the ECC and a vertex in the plane graph. When do ECCs yield witness lines?
Plane Graphs with ECCs

Degrees of different vertices

Witness line is a line associated with a change in the ECC and a vertex in the plane graph. When do ECCs yield witness lines?
Witness line is a line associated with a change in the ECC and a vertex in the plane graph. When do ECCs yield witness lines?
Plane Graphs with ECCs

Degrees of different vertices

*Witness line* is a line associated with a change in the ECC and a vertex in the plane graph. When do ECCs yield witness lines?
Plane Graphs with ECCs

Degrees of different vertices

Witness line is a line associated with a change in the ECC and a vertex in the plane graph. When do ECCs yield witness lines?
Plane Graphs with ECCs

Degrees of different vertices

Witness line is a line associated with a change in the ECC and a vertex in the plane graph. When do ECCs yield witness lines?
Plane Graphs with ECCs

Degrees of different vertices

Yes  No  No  Yes

Yes  Yes  No  No

Witness line is a line associated with a change in the ECC and a vertex in the plane graph. When do ECCs yield witness lines?
Degree two vertices are difficult to witness.
Plane Graphs with ECCs

Additional Problems
Degree two vertices can create three-way witness line intersections where no vertex exists...
Plane Graphs with ECCs

Additional Problems
Degree two vertices can create three-way witness line intersections where no vertex exists...
Plane Graphs with ECCs

Reconstruction

Reconstruction of vertices is possible under the following scenarios:

- No degree two vertices in graph.
  - Able to use similar method as Belton et al. (2018) but with twice as many ECCs as PDs (six vs three).
- If we know there are $n$ vertices we can create intersection points in $\mathbb{R}^2$ with $m > n$ witness lines (similar observation made in Lemma 7.1 of Curry et al. (2018)).
Outline

Talk Outline

- Motivation
- Background
- Previous Work and Problem Statement
- Previous Approach (Belton et al. (2018))
- Euler Characteristic Curves
- Summary and Future Work
Conclusion

Current Status

Summary

- Reconstruction using ECCs is difficult.
  - Degree two vertices difficult to witness and able to create unwarranted three-way intersections.
  - Lack of information about underlying graph.
- Algorithms for reconstruction exist with certain assumptions.
Previous Work

Citations

- Turner, K., Mukherjee, S., and Boyer, D. M. Persistent homology transform for modeling shapes and surfaces. Information and Inference: A Journal of the IMA 3, 4 (2014), 310–344
Questions?

Thanks!

Euler Characteristic

\[ s_1 \]

samuel.a.micka@gmail.com