

## CSCI 538 Computability

### Assignment 1 (10 marks)

#### Question 1 (2 marks)

Let  $c_1x^n + c_2x^{n-1} + \dots + c_nx + c_{n+1}$  be a polynomial with an integral root at  $x = x_0$ . Let  $c_{\max} = \max_{1 \leq i \leq n+1} |c_i|$ . Prove that

$$|x_0| < (n+1) \frac{c_{\max}}{|c_1|}.$$

#### Question 2 (2 marks)

Show that  $f(x, y) = \max\{x, y\}$ ,  $x, y \in \mathcal{I}$ , is primitive recursive.

#### Question 3 (2 marks)

Show that  $f(x, y) = x \bmod y$ ,  $x, y \in \mathcal{I}, y \neq 0$ , is primitive recursive.

#### Question 4 (2 marks)

Let  $ALL_{DFA} = \{ \langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^* \}$ . Show that  $ALL_{DFA}$  is decidable.

#### Question 5 (2 marks)

Let  $C_{CFG} = \{ \langle G, k \rangle \mid G \text{ is a context-free grammar and } L(G) \text{ contains exactly } k \text{ strings where } k \geq 0 \text{ or } k = \infty \}$ . Show that  $C_{CFG}$  is decidable.

**Date Due:** before the end of class on **Tuesday, Sep 15, 2015.**