Assignment 1 (10 marks)

Question 1 (2 marks)

Let $c_1 x^n + c_2 x^{n-1} + \cdots + c_n x + c_{n+1}$ be a polynomial with an integral root at $x = x_0$. Let $c_{\max} = \max_{1 \le i \le n+1} |c_i|$. Prove that

$$|x_0| < (n+1) \frac{c_{\max}}{|c_1|}.$$

Question 2 (2 marks)

Show that $f(x, y) = \max\{x, y\}$, $x, y \in \mathcal{I}$, is primitive recursive.

Question 3 (2 marks)

Show that $f(x, y) = x \bmod y$, $x, y \in \mathcal{I}, y \neq 0$, is primitive recursive.

Question 4 (2 marks)

Given a string $w \in \{0, 1\}^*$, w^R is its reversal. Let $S = \{< M > | M$ is a DFA that accepts w^R whenever it accepts $w\}$. Show that S is decidable.

Question 5 (2 marks)

Let $C_{CFG} = \{< G, k > | G$ is a context-free grammar and $L(G)$ contains exactly k strings where $k \geq 0$ or $k = \infty\}$. Show that C_{CFG} is decidable.

Date Due: before the end of class on **Thursday, Sep 14, 2017.**