CS324—Computer Science IV

Lecture 1

Principal Textbook: Fundamentals of sequential and parallel algorithms
   by Berman and Paul

Additional Textbook: Introduction to Algorithms
   by Cormen, Leiserson and Rivest
Discrete Mathematics
   by Rosen
0. About CS324

- Course home page: http://www.cs.montana.edu/bhz, or http://www.cs.montana.edu/bhz/classes/spring-2003/cs324

- We will cover advanced data structures and algorithms

- If you want detailed contents of the course, check the home page for last semester’s CS324 (http://www.cs.montana.edu/bhz/324-02). But the exact schedule of the two courses might be slightly different.

- Evaluation: in-class tests (30%), assignments (30%) and final exam (40%)

- Evaluation: in-class tests (30%), assignments (30%), best of the tests (10%) and final exam (30%)

- To pass the course, you must get at least 30 out of 100 in the final exam.
1. Overview

• Basic concepts on data structures and algorithms
• Quicksort, mergesort, heapsort
• Abstract data types, B-trees, heaps
• Greedy algorithms, divide-and-conquer, dynamic programming
• Randomization, exhaustive search, backtracking
• Shortest paths and Dijkstra’s algorithm
• Approximation algorithms, parallel algorithms, NP-completeness

• After finishing this course, students are expected to be able to solve application problems independently in other areas (courses). Under proper supervision and guidance, students are expected to be able to do some research and development work in the area of algorithm design and analysis.
2. Review of algorithmic complexity

- The measure of efficiency of a program (algorithm) $f(n)$ is called algorithmic complexity of $f(n)$.

- $O$-notation. $f(n)$ is $O(g(n))$ if there exist positive constants $c$ and $N$ such that $f(n) \leq c \cdot g(n)$ for all $n \geq N$.

- To follow the textbook, $O(g(n))$ is defined as the set of all functions $f(n)$ such that there exist positive constants $c, N$ and $f(n) \leq c \cdot g(n)$ for all $n \geq N$.

- Properties of $O$-notations
• \( \Omega \)-notation. \( \Omega(g(n)) \) is the set of all functions \( f(n) \) such that there exist positive constants \( c, N \) and \( f(n) \geq c \cdot g(n) \) for all \( n \geq N \).

• \( \Theta \)-notation. \( \Theta(g(n)) \) is the set of all functions \( f(n) \) such that there exist positive constants \( c_1, c_2, N \) and \( c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n) \) for all \( n \geq N \).
3. Establishing Order Relationships

- Notice that the notion of order is restricted to real-valued functions $f(n) : \mathbb{N} \rightarrow \mathbb{R}$ that are eventually positive; i.e., there exists an integer $n_0$ such that $f(n) > 0$ for all $n > n_0$. Let $\mathcal{F}$ be the set of such functions.

- Given $f(n), g(n) \in \mathcal{F}$, we say that $f(n)$ has smaller order than $g(n)$ if $O(f(n)) \subset O(g(n))$, i.e., $O(f(n))$ is strictly contained in $O(g(n))$.

- Let $P(n) = a_kn^k + a_{k-1}n^{k-1} + \ldots + a_1n + a_0$ be any polynomial of degree $k$, then $P(n) \in \Theta(n^k)$.
• Let \( f(n), g(n) \in \mathcal{F} \). Let the limit of \( \frac{f(n)}{g(n)} \) be \( L \) as \( n \to \infty \), i.e., \( L = \lim_{n \to \infty} \frac{f(n)}{g(n)} \), then the following results hold.

  - If \( 0 < L < \infty \), then \( f(n) \in \Theta(g(n)) \).
  - If \( L = 0 \), then \( O(f(n)) \subset O(g(n)) \).
  - If \( L = \infty \), then \( O(g(n)) \subset O(f(n)) \).

• Let \( f(n), g(n) \in \mathcal{F} \). We define \( o(g(n)) \) as the set of all functions \( f(n) \) such that \( \lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \).
• Let $f(n), g(n) \in \mathcal{F}$. We say $f(n)$ is strongly asymptotic to $g(n)$ if $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 1$. (Certainly $f(n) \in \Theta(g(n))$.)

• L’Hôpital’s Rule: Let $f(x)$ and $g(x)$ be functions that are differentiable for sufficiently large real numbers $x$. If $\lim_{x \to \infty} f(x) = \infty$ and $\lim_{x \to \infty} g(x) = \infty$, then

\[
\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{f'(x)}{g'(x)}.
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