Assignment 3 (10 marks)

Question 1 (1 mark)

Decide whether the following grammar is ambiguous.

\[
S \rightarrow AB | aaB \\
A \rightarrow a | Aa \\
B \rightarrow b
\]

Question 2 (1 mark)

Convert the following CFG G to an equivalent PDA.

\[
R \rightarrow XRX | S \\
S \rightarrow aTb | bTa \\
T \rightarrow XTX | X | \epsilon \\
X \rightarrow a | b
\]

Question 3 (1 mark)

Let \(G = (V, \Sigma, R, S)\) be the following grammar. \(V = \{S, T, U\}\); \(\Sigma = \{0, \#\}\); and \(R\) is the set of rules:

\[
S \rightarrow TT | U \\
T \rightarrow 0T | T0 | \# \\
U \rightarrow 0U00 | \#
\]

(3.1) Describe \(L(G)\) in English.

(3.2) Prove that \(L(G)\) is not regular.

Question 4 (1 mark)

Convert the following CFG into an equivalent CFG in Chomsky Normal Form

\[
A \rightarrow BAB | B | \epsilon \\
B \rightarrow 00 | \epsilon
\]

Question 5 (2 marks)

Using pumping lemma to prove that the following languages are not context-free.

(5.1) \(L = \{a^n b^j c^k | k = nj\}\).

(5.2) \(L = \{a^n b^n | n \geq (j - 1)^3\}\).
Question 6 (1 mark)

Let $B$ be the set of all infinite sequences over $\{a, b\}$. Show that $B$ is uncountable, using a proof by diagonalization.

Question 7 (1 mark)

Let $A = \{⟨R, S⟩|R$ and $S$ are regular expressions and $L(R) \subseteq L(S)\}$. Show that $A$ is decidable.

Question 8 (1 mark)

Let $\Sigma = \{a, b\}$. Define the following language $ODD_{TM}$:

$ODD_{TM} = \{< M > | M$ is a TM and $L(M)$ contains only strings of odd length $\}$.

Prove that $ODD_{TM}$ is undecidable.

Question 9 (1 mark)

Show that $EQ_{CFG}$ is undecidable.

Date Due: before the end of class on Friday, March 27, 2015. No late assignment will be accepted.