

CSCI 538 Computability

Assignment 1 (10 marks)

Question 1 (2 marks)

Let $c_1x^n + c_2x^{n-1} + \dots + c_nx + c_{n+1}$ be a polynomial with an integral root at $x = x_0$. Let $c_{\max} = \max_{1 \leq i \leq n+1} |c_i|$. Prove that

$$|x_0| < (n+1) \frac{c_{\max}}{|c_1|}.$$

Question 2 (2 marks)

Show that $f(x, y) = \max\{x, y\}$, $x, y \in \mathcal{I}$, is primitive recursive.

Question 3 (2 marks)

Show that $f(x, y) = x \% y$, $x, y \in \mathcal{I}, y \neq 0$, is primitive recursive. ($x \% y$ is the remainder calculation operation, e.g., $2 \% 4 = 2$, $7 \% 3 = 1$.)

Question 4 (2 marks)

Given a string $w \in \{0, 1\}^*$, w^R is its reversal. Let $S = \{ \langle M \rangle \mid M \text{ is a DFA that accepts } w^R \text{ whenever it accepts } w \}$. Show that S is decidable.

Question 5 (2 marks)

Let $C_{CFG} = \{ \langle G, k \rangle \mid G \text{ is a context-free grammar and } L(G) \text{ contains exactly } k \text{ strings where } k \geq 0 \text{ or } k = \infty \}$. Show that C_{CFG} is decidable.

Date Due 8:30pm, Tuesday, Feb 6, 2024. You should upload your assignment in pdf format on D2L, under the “Assignment 1” directory.