Weak Kernels and Their Applications

Binhai Zhu

Department of Computer Science
Montana State University
Bozeman, MT 59717-3880
USA
Email: bhz@cs.montana.edu

Traditional NP-completeness theory [3, 8, 7] and FPT algorithms theory [4, 6] focus on *decision* problems. Valiant first raised the question on whether *search* problems and decision problems are equivalent [9], which leads to a small branch in traditional complexity theory. The notable conclusion is that even for problems in NP, they are not equivalent, i.e., under a complexity assumption, there is an associated search problem ρ in NP which cannot be reduced to its corresponding decision problem [2].

In the FPT theory, *kernelization* (also viewed as *data reduction*), is about reducing problem size and is one of the most fundamental techniques in the area [4, 5]. In this work, we formalize a folklore concept and formally define *weak kernels* for (NP-hard) *search* problems, which is about search space reduction and stands as a new generic technique for designing FPT algorithms. (The idea of designing FPT algorithms for search problems was used by Bosma et al. as early as in 2003 [1].) We show that weak kernels are different from the (traditional) kernels for decision problems, by exhibiting an example out of P such that its decision version has no kernel while the equivalent search problem has a weak kernel.

As this work was motivated by applications in computational biology, we show a few applications of weak kernels on some of these problems, for which a traditional kernelization seems hard to apply. The three applications are: Sorting by Minimum Unsigned Reversals, Minimum Co-Path Set and Sorting by Minimum Unsigned DCJ Distance. Among them, we present the first FPT algorithm for the famous Sorting by Minimum Unsigned Reversals problem. While the three applications are mainly from genome analysis, hopefully new applications will follow later on.

Acknowledgments This is a joint work with Haitao Jiang and Chihao Zhang. This research is partially supported by NSF of China under project 60928006 and by NSF under project DMS-0918034. We also thank Yijia Chen, Mike Fellows and Angsheng Li for several valuable comments.

References

1. P. Bonsma, T. Brüggemann and G. Woeginger. A fast FPT algorithm for finding spanning trees with many leaves. In *Proc. 28th Intl. Symp. on Mathematical Foundations of Computer Science (MFCS'03), pages 259-268, 2003.*
2. M. Bellare and S. Goldwasser. The complexity of decision versus search. *SIAM J. Comput.*, 23:97–119, 1994.
3. S. Cook. The complexity of theorem-proving procedures. In *Proceedings of the 3rd ACM Symp. on Theory of Computing (STOC'71)*, pages 151-158, 1971.
4. R. Downey and M. Fellows. *Parameterized Complexity*, Springer-Verlag, 1999.
5. M. Fellows. The lost continent of polynomial time: preprocessing and kernelization. In *Proc. 2nd Intl. Workshop on Parameterized and Exact Computation (IWPEC'06)*, LNCS 4169, pages 276-277, 2006.
6. J. Flum and M. Grohe. *Parameterized Complexity Theory*, Springer-Verlag. 2006.
7. M. R. Garey and D. S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman, 1979.
8. R. Karp. Reducibility among combinatorial problems. In R. Miller and J. Thatcher (eds.), *Complexity of Computer Computations*, Plenum Press, NY, pages 85–103, 1972.
9. L. Valiant. On the relative complexity of checking and evaluating. *University of Leeds Technical Report LS29JT, October*, 1974.