Computer Networks 52 (2008) 2148–2158

Contents lists available at ScienceDirect

Computer Networks

journal homepage: www.elsevier.com/locate/comnet

Joint spectrum allocation and scheduling for fair spectrum sharing in cognitive radio wireless networks ☆

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ARTICLE INFO

Article history:
Received 21 December 2007
Received in revised form 28 February 2008
Accepted 9 March 2008
Available online 6 April 2008

Responsible Editor: Dr. J. Misic

Keywords:
Cognitive radio
Dynamic spectrum access
Spectrum allocation
Scheduling
Fairness

ABSTRACT

Cognitive radio and Dynamic Spectrum Access (DSA) enable wireless users to share a wide range of available spectrums. In this paper, we study joint spectrum allocation and scheduling problems in cognitive radio wireless networks with the objectives of achieving fair spectrum sharing. A novel Multi-Channel Contention Graph (MCCG) is proposed to characterize the impact of interference under the protocol model in such networks. Based on the MCCG, we present an optimal algorithm to compute maximum throughput solutions. As simply maximizing throughput may result in a severe bias on resource allocation, we take fairness into consideration by presenting optimal algorithms as well as fast heuristics to compute fair solutions based on a simplified max–min fairness model and the well-known proportional fairness model. Numerical results show that the performance given by our heuristic algorithms is very close to that of the optimal solution, and our proportional fair algorithms achieve a good tradeoff between throughput and fairness. In addition, we extend our research to the physical interference model, and propose effective heuristics for solving the corresponding problems.

1. Introduction

Over the past few years, the world has experienced a very rapid proliferation of wireless devices. The traditional static spectrum access approach, which assigns a fixed portion of the spectrum to a specific license holder or a wireless service for exclusive use, is unable to manage the spectrum efficiently any longer. On one hand, certain parts of the spectrum are heavily used, such as the 2.4 GHz band and the 5 GHz band, which leads to serious interference and therefore poor network throughput. On the other hand, a significant amount of spectrums remain under-utilized or not utilized at all, which has been shown by recent studies and experiments [2].

The most efficient and direct method to solve the above problems is to allow wireless users to share a wide range of available spectrums. Emerging cognitive radio technology and the Dynamic Spectrum Access (DSA) approach enable unlicensed wireless users (a.k.a secondary users) to sense and access the under-utilized spectrum opportunistically even if it is licensed, as long as the licensed wireless users (a.k.a primary users) in such a spectrum band are not interfered. A network composed of wireless users with cognitive radios and dynamic spectrum access capabilities is called a cognitive radio wireless network or a DSA wireless network [2].

How to efficiently and fairly share the available spectrums is a fundamental and challenging problem in cognitive radio wireless networks [2]. In a multihop wireless network, a wireless user usually refers to a transmitter and receiver pair (a wireless link) [11]. The spectrum sharing problem usually involves two coupled problems: the

☆ This research was supported in part by NSF Grants CNS-0721880, CNS-0721803, CCF-0431167 and ARO Grant W911NF-04-1-0385. The information reported here does not reflect the position or the policy of the federal government.

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doi:10.1016/j.comnet.2008.03.010

spectrum allocation problem and the scheduling problem. The spectrum allocation problem seeks a solution which allocates available spectrum bands to the users for packet transmissions. The scheduling problem looks for a solution which determines when these users can access the allocated spectrum bands. The objective is to achieve a good tradeoff between throughput and fairness while ensuring interference-free transmission at any time. In this paper, we present optimal algorithms as well as fast heuristic algorithms to solve the joint spectrum allocation and scheduling problems in multihop cognitive radio wireless networks. Specifically, our contributions are summarized as follows:

1. We propose a novel *Multi-Channel Contention Graph (MCCG)* to precisely characterize the impact of interference in a cognitive radio wireless network.
2. We study the joint spectrum allocation and scheduling problems, which have never been seriously addressed before in the context of multihop cognitive radio wireless networks. We present optimal algorithms as well as fast heuristic algorithms to solve these problems and evaluate their performance by extensive simulations.
3. We take account of both the protocol and the physical interference models [7], making our solutions more comprehensive and more suitable for practical scenarios. If each wireless user is assumed to transmit at a fixed power level, the protocol model can be used to address interference. However, if users have the power control capability, the physical interference model should be considered.

The rest of this paper is organized as follows. We discuss related work in Section 2. The system model is described in Section 3. We define the problems to be studied in Section 4. The proposed spectrum allocation and scheduling algorithms are presented in Section 5. We present numerical results in Section 6 and conclude the paper in Section 7.

2. Related work

The cognitive radio wireless networks have recently attracted lots of research attention. The most related work is [26], in which Zheng et al. developed a graph-theoretic model to characterize the spectrum access problem and devised a set of heuristics to find high throughput and fair solutions. In [24], the concept of a time-spectrum block was introduced to model spectrum reservation, and protocols were presented to allocate such blocks. A centralized spectrum allocation protocol called Dynamic Spectrum Access Protocol (DSAP) was proposed in [5]. In DSAP, spectrum management is conducted in a central entity called DSAP server which can obtain a global view of network by exchanging information with users. In [6], a distributed spectrum allocation scheme based on local bargaining was proposed for cognitive radio wireless ad hoc networks. In [25], the authors derived optimal and suboptimal distributed strategies for the secondary users to decide which channels to sense and access with the objective of throughput maximization under a framework of Partially Observable Markov Decision Process (POMDP).

Cross-layer schemes have also been proposed for cognitive radio wireless networks. In [20], Wang et al. considered the joint design of dynamic spectrum access and adaptive power management. They proposed a power-saving multi-channel MAC protocol (PSM-MMAC), which is capable of reducing the collision probability and the waiting time in the awake state of a node. The authors of [11] proposed the Asynchronous Distributed Pricing (ADP) scheme to solve a joint spectrum allocation and power assignment problem. In [23], a novel layered graph was proposed to model spectrum access opportunities, which was used to develop joint spectrum allocation and routing algorithms. In [21], two design methodologies were explored: a decoupled (layered) design and a collaborative (cross-layer) design. The authors implemented the idea of collaborative design by proposing joint routing, scheduling and spectrum allocation algorithms. A Mixed Integer Non-Linear Programming based algorithm was presented to solve a joint spectrum allocation, scheduling and routing problem in [10]. In addition, the authors of [22] presented distributed algorithms for joint spectrum allocation, power control, routing, and congestion control.

Maximum throughput and fair resource allocation (channel assignment, scheduling) has also been studied for traditional multihop wireless mesh networks in [1,18,19]. The differences between this work and previous works are summarized as follows: First of all, resource allocation in a cognitive radio wireless network is quite different from that in traditional multihop wireless networks such as 802.11-based wireless mesh networks due to its special features such as dynamic channel availability, channel heterogeneity and so on (refer to [2] for details). Second, fairness is a major concern of this work. However, the schemes proposed in [10,20,21,23,25] achieve different optimization goals such as minimizing power consumption, maximizing throughput and minimizing bandwidth usage. Third, this paper focuses on the joint spectrum allocation and scheduling problems. However, scheduling has not been well addressed by [5,6,11,21–23,25,26]. Fourth, we propose algorithms to optimally solve the formulated problems. However, only heuristic algorithms were proposed in [5,6,10,11,20,21,23,26], which cannot provide any performance guarantees. In addition, we consider both the protocol and physical interference models. However, in most of previous works on spectrum allocation [5,6,21,23,24,26], only the protocol interference model has been considered.

3. System model

We consider a multihop cognitive radio wireless network composed of static secondary *users*, each of which refers to *a transmitter and receiver pair (i.e., a wireless link)*. The network can be either a traditional single radio wireless network or an emerging multi-radio wireless network [16] in which each node is equipped with multiple transceivers. The available spectrums are divided into a set of orthogonal spectrum bands, which are also called *channels*. We assume that a user can dynamically access a channel to deliver its packets, but can only work on one of the available channels at one time. Any proposed spectrum sensing schemes [2] can be used to detect the locally available channels. Half-duplex operation is assumed to prevent self-interference, i.e., one transceiver can only transmit or receive at one time. Moreover, we only consider unicast communication, i.e., a single transmission is intended for exactly one receiver. In addition, any two transmissions with a common intended receiver are not allowed to be made simultaneously since collisions will corrupt the packet receptions. We say a user (link) is *incident* to another user if they share a common transceiver. We also say a user is incident to itself (this is a technical agreement which will make future description easier).

We address wireless interference based on both the protocol model and the physical model [7]. In a multi-channel network, interference should be defined on user-channel pairs. In the protocol model, it is assumed that each transmitter transmits at a fixed transmission power. So there is a fixed transmission range and a fixed interference range (which is typically 2–3 times of the transmission range [16]) associated with each user. These two ranges may vary with the channels [26]. Two user-channel pairs (i,j) and (k,h) are said to interfere with each other if (1) user i is incident to user k, or (2) $j=h$ and $d(T(i),R(k)) \leqslant I_i^j$ or $d(T(k),R(i)) \leqslant I_k^h$, where $T(i)$ and $R(i)$ represent the transmitter and the receiver of user i respectively, $d(\cdot)$ gives the Euclidean distance between two nodes, and I_i^j denotes the interference range of user i on channel j. Condition (2) implicitly covers the constraints enforced by half-duplex operation, unicast communication and collision. However, if user i is incident to user k and even if $j \neq h$, we say user-channel pairs (i,j) and (k,h) interfere with each other, since two incident users cannot work on different channels at the same time. This case is not covered by condition (2) and is the reason for having condition (1). If two user-channel pairs interfere with each other, they cannot be active simultaneously, otherwise the corresponding transmissions will fail.

Let τ_j be the set of concurrent user-channel pairs with the same channel j and user-channel pair $(i,j) \in \tau_j$, then transmissions on user i over channel j can be successful if

$$\frac{G^j_{T(i)R(i)}P_i}{N_0 + \sum_{(k,j)\in\tau_j\setminus\{(i,j)\}} G^j_{T(k)R(i)}P_k} \geqslant \beta, \tag{1}$$

where $G^j_{T(i)R(i)}$ is the channel gain for the transmitter and the receiver of user i on channel j, which depends on path loss, channel fading and shadowing; P_i is the power level at the transmitter of user i; N_0 is the thermal noise power at the receiver of user i which is normally a constant. The left hand side of the inequality is called the *Signal to Interference and Noise Ratio (SINR)* at the receiver of user i and β is a given threshold determined by certain physical layer Quality of Service (QoS) requirements such as *Bit Error Rate (BER)*. This is introduced in [7] as the physical model for concurrent wireless transmissions. Here, we assume that each user transmits at a fixed rate on a specific channel even if it can adjust its transmission power.

Similar as in [5], a spectrum server is assumed to manage the spectrum allocation and scheduling in the network. It can collect information (including traffic demand and channel availability information) from all users periodically. Based on the collected information, the server computes a spectrum allocation and scheduling solution and broadcasts it to all the users at the beginning of each scheduling period. All the users will then access the spectrum according to the received solution. The control messages may also be exchanged over a common control channel using an extra control radio (no need to be a cognitive radio) if they are available for each node. In this case, the cognitive radios are only used for data transmission which can be conducted concurrently with the control information exchange. The server recomputes the scheduling and channel allocation solution whenever it finds out that the channel availability or traffic demands change.

4. Problem definition

In this section, we will describe the necessary notations and formally define the optimization problems to be studied.

Suppose that we are given a set of N users indexed from 1 to N and a set of C channels indexed from 1 to C. Then we can identify the set of possible user-channel pairs, denoted as $\mathscr{A}$. Here, a user-channel pair (i,j) is in $\mathscr{A}$ if and only if channel j is available to user i. The total number of user-channel pairs is bounded by $N \cdot C$. We are also given a vector $\mathbf{d} = [d_1, d_2, \ldots, d_N]$, specifying the traffic demand of each user, which is determined by a routing algorithm in the network layer. However, routing is out of scope of this paper.

We introduce the notion *transmission mode* to assist the computation. A transmission mode is composed of a subset of user-channel pairs which can be active concurrently. Whether concurrent transmissions are allowed or not can be determined based on the interference models described in the last section. Since every element of a transmission mode is a user-channel pair, once a transmission mode is identified, a spectrum allocation is automatically determined for the set of users contained in those user-channel pairs. We employ a $T \times M$ matrix Γ to represent the set of transmission modes, where M is the total number of possible user-channel pairs, and T is the number of transmission modes. Each row of the matrix corresponds to a transmission mode and each column corresponds to a specific user-channel pair in $\mathscr{A}$. If transmission mode t includes user-channel pair (i,j), then $\Gamma^t_{ij} = 1$. Otherwise, $\Gamma^t_{ij} = 0$. For ease of presentation, we always append a special all-zero row at the end of Γ which represents a transmission mode that does not contain any user-channel pair.

The average data rate of user i can be computed as $\sum_{j=1}^{C}\sum_{t:\Gamma^t_{ij}=1} p_t c_i^j$, where p_t is the fraction of time that transmission mode t is activated and c_i^j is the capacity of user (link) i on channel j which is usually a constant. In a scheduling-based wireless system, there will be a specific transmission mode activated for each time slot. Suppose that all possible transmission modes are given. The scheduling problem is to determine the frame length and the number of active time slots of each transmission mode in one frame. If the value of p_t is computed for each transmission mode, a frame length can be easily determined by finding the smallest positive integer L such that $p_t \cdot L$ is an integer for each transmission mode.

In this way, the joint spectrum allocation and scheduling problem is transformed into a problem of finding all possible transmission modes and the active time fraction for each transmission mode. In our optimization problems, we seek a rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_i, \ldots, r_N]$ which specifies the rate r_i allocated to each user i, all possible transmission modes along with a transmission schedule vector $\mathbf{p} = [p_1, p_2, \ldots, p_t, \ldots, p_T]$ which specifies the active time fraction p_t for each transmission mode t. A rate allocation vector and a transmission schedule vector are said to be *feasible* if the rate allocated to each user is no more than the average link data rate which can be achieved by the corresponding transmission schedule vector.

Now we are ready to define the joint spectrum allocation and scheduling problems.

Definition 1 (*MASS*). The MAximum throughput Spectrum allocation and Scheduling (MASS) problem seeks a feasible rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_N]$, all transmission modes along with a feasible transmission schedule vector such that the throughput $\sum_{i=1}^{N} r_i$ is maximized.

It has been shown that simply maximizing throughput may seriously starve some users in the network [9]. So fairness must be carefully addressed. The traffic demands for users may be quite different. Hence, addressing fairness simply based on the value of rate allocated to each user without taking into account its traffic demand is not a good idea. We define a new variable called Demand Satisfaction Factor (DSF). The DSF of a user is defined as the ratio of the rate allocated to that user over its traffic demand, which indicates how much a traffic demand is satisfied according to a rate allocation vector. Therefore, we will have a DSF vector $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_i, \ldots, \alpha_N]$ corresponding to each rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_i, \ldots, r_N]$, where $\alpha_i = r_i/d_i$, $1 \leqslant i \leqslant N$. The fair spectrum allocation and scheduling problems are defined as follows.

Definition 2 (*MMASS*). A feasible rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_N]$ $(\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_N])$ is said to be a feasible max–min fair rate allocation vector if for any other feasible rate allocation vector $\mathbf{r}' = [r'_1, r'_2, \ldots, r'_N]$ $(\alpha' = [\alpha'_1, \alpha'_2, \ldots, \alpha'_N])$, $\min\{\alpha_i | 1 \leqslant i \leqslant N\} \geqslant \min\{\alpha'_i | 1 \leqslant i > \leqslant N\}$, where α and α' are the DSF vectors corresponding to $\mathbf{r}$ and $\mathbf{r}'$ respectively. The Max–min fair MAximum throughput Spectrum allocation and Scheduling (MMASS) problem seeks a feasible max–min fair rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_N]$, all transmission modes along with a feasible transmission schedule vector such that the throughput $\sum_{i=1}^{N} r_i$ is maximized.

Definition 3 (*PASS*). The Proportional fAir Spectrum allocation and Scheduling (PASS) problem seeks a feasible rate allocation vector $\mathbf{r} = [r_1, r_2, \ldots, r_N]$ $(\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_N])$, all transmission modes along with a feasible transmission schedule vector such that the utility function $\sum_{i=1}^{N} \log(\alpha_i)$ is maximized, where α is the DSF vector corresponding to $\mathbf{r}$.

So far, we have only defined the joint spectrum allocation and scheduling problems under the protocol interference model. The corresponding optimization problems under the physical interference model are almost the same as their counterparts under the protocol model except that a feasible power assignment needs to be determined for each transmission mode. By feasible, we mean that on each channel, the SINR constraint (constraint (1)) must be satisfied at each receiver and the power level assigned to each user must be in the range of $[0, P_{\max}]$. Due to the space limit and redundancy, we omit the corresponding problem definitions.

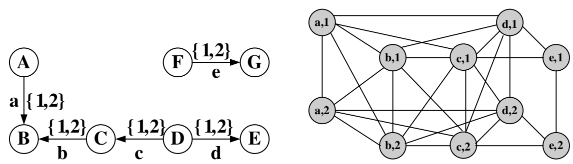
5. Proposed spectrum allocation and scheduling algorithms

In this section, we will first introduce a novel graph model, Multi-Channel Contention Graph (MCCG), to characterize the impact of interference under the protocol model. Based on it, we will present algorithms to solve the problems defined in Section 4. Then we will discuss the extension to the physical interference model.

5.1. Multi-channel contention Graph (MCCG)

In an MCCG $G_C(V_C, E_C)$, every vertex corresponds to a user-channel pair in $\mathscr{A}$. There is an undirected edge connecting two nodes in V_C if their corresponding user-channel pairs *interfere* with each other, which can be determined based on conditions described in Section 3. Note that if two users i, k are incident to each other, then there will be undirected edges between every two user-channel pairs which contain i and k, respectively because they always interfere with each other no matter which channels are considered.

Next, we use a simple example to illustrate how to construct an MCCG. In this example, we have five users (transmitter–receiver pairs), a, b, c, d, e, and two channels, channel 1 and channel 2, available to each user, which are shown in Fig. 1a. In the figure, we have $d(A,B) = d(B,C) = d(C,D) = d(D,E) = d(F,G) = d(D,F) = d(E,G) = R = 0.5I$, where R and I are the transmission and interference range of each user respectively. We can obtain the corresponding MCCG which is shown in Fig. 1b. In the figure, each vertex corresponds to a user-channel pair, for example, vertex $(a,2)$ corresponds to user-channel pair $(a,2)$. Here, we can see that there are edges between nodes $(a,1)$ and $(b,1)$, $(a,1)$ and $(b,2)$, $(a,2)$ and $(b,1)$, and $(a,2)$ and $(b,2)$, because user a is incident to



(a) A set of users (b) The corresponding MCCG

Fig. 1. MCCG: (a) a set of users; (b) the corresponding MCCG.

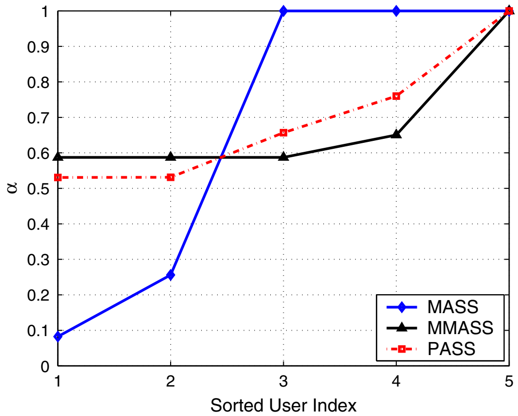
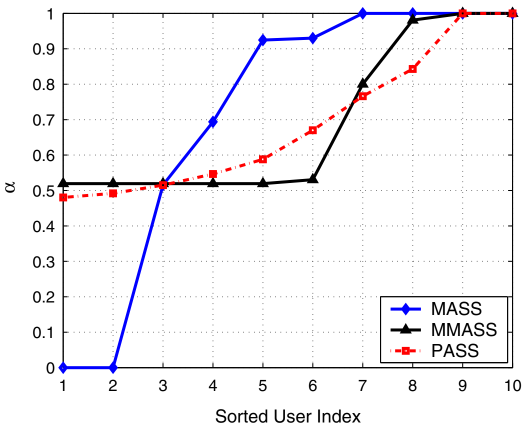


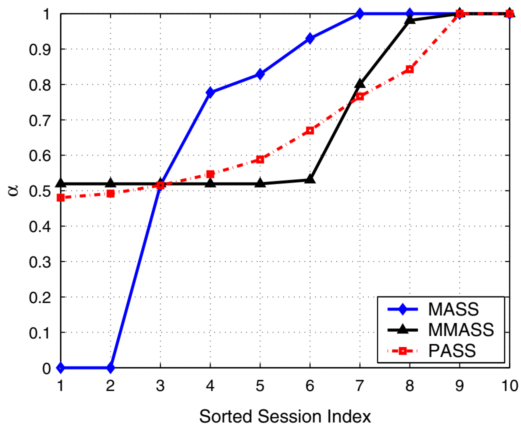
Fig. 2. Scenario 1: protocol model with $N = 5$ and $C = 2$.

user *b*. Moreover, there is an edge between node $(a, 1)$ and $(a, 2)$ because any user can only work on one channel at one time.

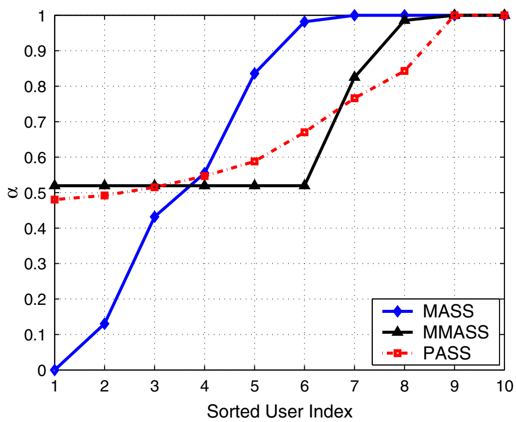
The importance of the MCCG lies in the fact that a transmission mode corresponds to an independent set in the MCCG. Since our objective is to improve throughput and fairness, we only need to consider the subset of transmission modes corresponding to Maximal Independent Sets (MISs) of G_C. The MCCG turns out to be a very useful tool for spectrum allocation in cognitive radio networks with multiple channels. For example, it can be used to find all possible transmission modes for our joint spectrum allocation and scheduling problems. In addition, the Max-Sum-Bandwidth (MSB) spectrum allocation problem studied in [26] can be transformed to the maximum weight independent set problem on the MCCG, which can be efficiently solved by some approximation algorithms in the literature [8]. Note that the MCCG is an extension of the well-known contention graph proposed in [15] for single-channel wireless networks and it is completely different from the other graph models introduced for multi-channel wireless networks [23,26].



(a) Scenario 2: optimal algorithms



(b) Scenario 3: heuristic algorithms with $q = 1$



(c) Scenario 4: heuristic algorithms with $q = 2$

Fig. 3. Protocol model with $N = 10$, $C = 6$ and $d_i = [7.2, 16.8]$.

5.2. Proposed algorithms for the protocol model

Our algorithms are essentially two-step methods: in the first step, construct the MCCG and identify transmission modes; in the second step, formulate the problems defined in Section 4 as Linear Programming (LP) or Convex Programming (CP) problems, and solve them using existing algorithms [3,4].

If a set of transmission modes is given, the MASS problem and the MMASS problem can be formulated as LPs, and the PASS problem can be formulated as a CP, which will be shown later. If the given set includes all possible transmission modes, then by solving those LPs and CP, we can obtain optimal solutions. Otherwise, if the given set only include a subset of all transmission modes, then we will end up with approximate solutions. In the rest of this section, we will first present algorithms to find a set of transmission modes and then present LP and CP formulations for the three optimization problems.

As discussed before, a transmission mode actually corresponds to an independent set in the MCCG and only those MISs are needed to be taken into consideration. So, if we can identify all possible MISs in the MCCG, then we can obtain optimal solutions. The algorithm in [14] and several other existing algorithms can actually find all MISs

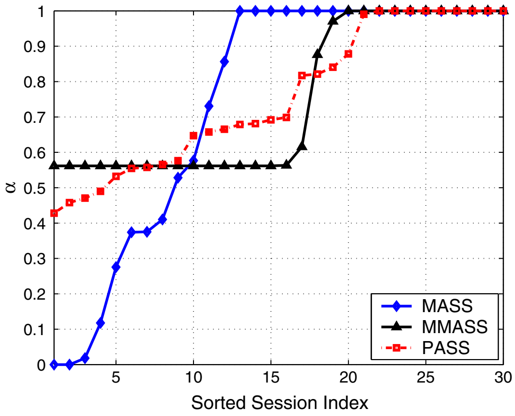
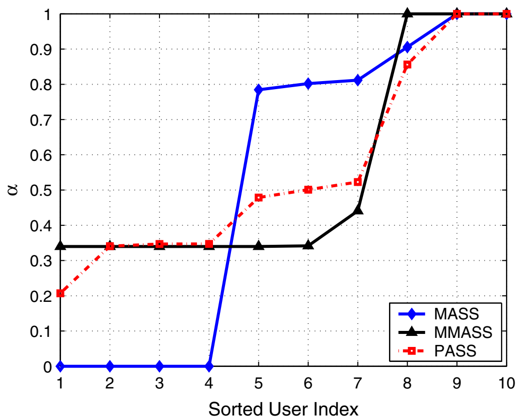
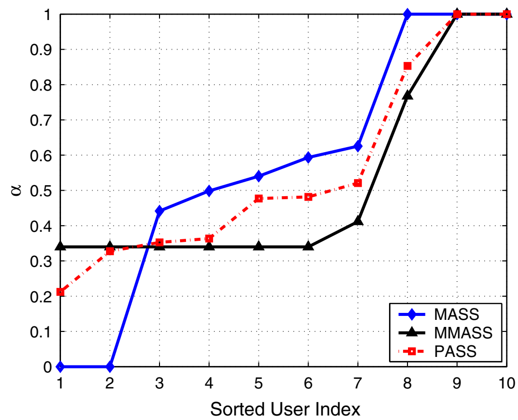


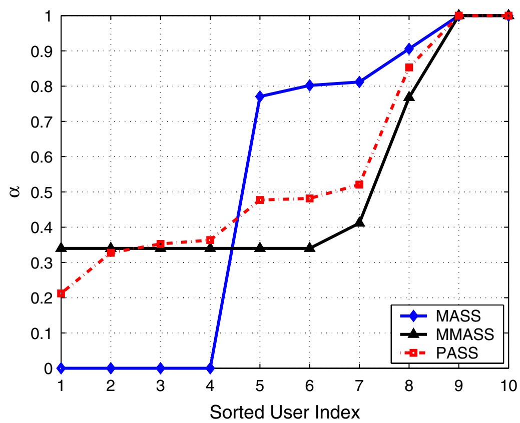
Fig. 5. Scenario 8: protocol model with $N = 30$, $C = 12$ and $d_i = [7.2, 16.8]$.



(a) Scenario 5: optimal algorithms



(b) Scenario 6: heuristic algorithms with $q = 1$



(c) Scenario 7: heuristic algorithms with $q = 2$

Fig. 4. Protocol model with $N = 10$, $C = 6$ and $d_i = [12, 24]$.

in a graph very efficiently. Therefore, our *optimal algorithm* for the MASS problem is to apply the algorithm in [14] to find all MISs in the MCCG firstly and then solve the corresponding LP. Similarly, we can have the *optimal algorithms* for the MMASS and PASS problems.

However, it is well known that the number of all MISs in a graph may grow exponentially with the graph size. If we take all MISs as the inputs for the LPs and the CP, it may take exponentially long time to solve them. Therefore, we propose a polynomial time heuristic to compute a *good* subset of MISs (transmission modes) in a given MCCG. Intuitively, a good subset needs to have good diversity, because if only a small subset of user-channel pairs is included, it may lead to biased solutions in the second step. Furthermore, the user-channel pair whose transmission capacity is relatively large and whose corresponding user has relatively high traffic demand should be given higher priority. Our algorithm is formally presented as Algorithm 1.

Algorithm 1. Computing transmission mode subset	
Step 1	$\mathscr{T} := \emptyset$; $i := 1$; $X[v] := 0$; **forall** $v \in V_C$

Step 2	**while** $(i <= q)$
	forall $v \in V_C$
	$S := \emptyset$; Add v to S;
	$X[v] := X[v] + 1$;
	do Add node $u \neq v$ to S, s.t. u has maximum weight $w(u) = (d_{\pi(u)} c_u)/(X[u] + 1)$ among all nodes which is not identical or incident to any other existing node in S;
	$X[u] := X[u] + 1$;
	until S becomes an MIS;
	if $(S \notin \mathscr{T})$
	$\mathscr{T} := \mathscr{T} \cup \{S\}$;
	endif
	endforall
	$i := i + 1$;
	endwhile

Step 3	**output** $\mathscr{T}$;

In Algorithm 1, set S is used to record an MIS computed during the execution of the algorithm. $\mathscr{T}$ is output as the subset of all transmission modes and is guaranteed to cover every node in G_C at least once due to Step 2. Array X is used to maintain a counter which counts how many times a node has been included in some MISs of $\mathscr{T}$ so far. The weight of each node v in G_C is given as $w(v) = (d_{\pi(v)} c_v)/(X[v] + 1)$, where $\pi(v)$ gives the corresponding user of node v (note that every node in G_C corresponds to a user-channel pair) and $d_{\pi(v)}$ gives its traffic demand. c_v is the capacity of the user-channel pair corresponding to v. The weight function $w(\cdot)$ implements the idea that we prefer to select the user-channel pair whose transmission capacity is relatively large and whose corresponding user have relatively high traffic demand. Moreover, based on the weight function, if the number of times a user-channel pair is covered is relatively small, it will get more chances to be selected. In this way, a good selection diversity can be achieved. In the algorithm, q is a tunable parameter. We observe that the larger the value of q is, the more MISs will be added into $\mathscr{T}$, which will lead to better solutions but longer computation time. Obviously, Algorithm 1 is a polynomial time algorithm. Its running time is dominated by Step 2, which can be accomplished in $O(q^2 M + qM^3)$ time, where M is the total number of possible user-channel pairs which is bounded by $N \cdot C$.

After obtaining a set of transmission modes, we can solve the optimization problems defined above by solving an LP or a CP, which are presented as follows. In the following formulations, we have the aforementioned rate allocation variables r_i or α_i to represent the rate or the DSF of user i respectively, and the scheduling variables p_t. The feasibility of rate allocation and scheduling described in Section 4 are guaranteed by constraint (3) or (8) which are actually equivalent. The summation of all scheduling variables should be equal to 1, which is ensured by constraint (4). Only non-negative values are allowed for all those variables and the value of α_i must be in $[0, 1]$, which are enforced by constraints (5), (6) and (11). User-channel pair capacity (c_i^j) and traffic demand of each user (d_i) are given as inputs.

LP1: MASS

$$\max \quad \sum_{i=1}^{N} r_i \tag{2}$$

subject to:

$$r_i \leqslant \sum_{j=1}^{C} \sum_{t:\Gamma_{ij}^t = 1} p_t c_i^j, \quad 1 \leqslant i \leqslant N; \tag{3}$$

$$\sum_{t=1}^{T} p_t = 1; \tag{4}$$

$$p_t \geqslant 0, \quad 1 \leqslant t \leqslant T; \tag{5}$$

$$0 \leqslant r_i \leqslant d_i, \quad 1 \leqslant i \leqslant N. \tag{6}$$

LP2: Max–min δ

$$\max \quad \delta \tag{7}$$

subject to:

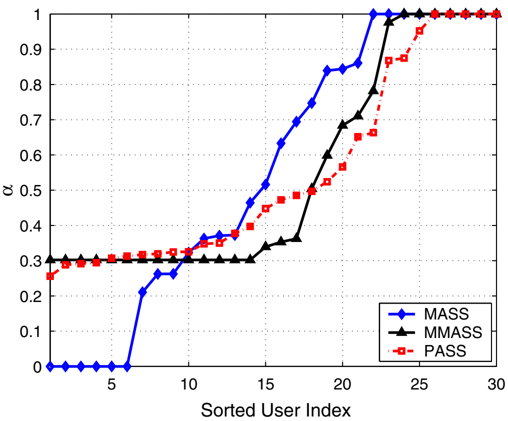


Fig. 6. Scenario 9: protocol model with $N = 30$, $C = 12$ and $d_i = [12, 24]$.

$$\alpha_i d_i \leqslant \sum_{j=1}^{C} \sum_{t:\Gamma_{ij}^t=1} p_t c_i^j, \quad 1 \leqslant i \leqslant N; \tag{8}$$

$$\sum_{t=1}^{T} p_t = 1;$$
$$p_t \geqslant 0, \quad 1 \leqslant t \leqslant T;$$
$$\delta \leqslant \alpha_i \leqslant 1, \quad 1 \leqslant i \leqslant N. \tag{9}$$

LP3(δ): MMASS

$$\max \quad \sum_{i=1}^{N} r_i$$

subject to:

$$r_i \leqslant \sum_{j=1}^{C} \sum_{t:\Gamma_{ij}^t=1} p_t c_i^j, \quad 1 \leqslant i \leqslant N;$$
$$\sum_{t=1}^{T} p_t = 1; \tag{10}$$
$$p_t \geqslant 0, \quad 1 \leqslant t \leqslant T;$$
$$\delta d_i \leqslant r_i \leqslant d_i, \quad 1 \leqslant i \leqslant N.$$

The MASS problem can be solved by solving *LP*1 in which the objective function is set to maximize the throughput. The maximum throughput solution can serve as a benchmark to evaluate the fair solutions provided by solving the corresponding MMASS and PASS problems. In order to solve the MMASS problem, we need to solve two *LPs* sequentially. First, we solve *LP*2 and obtain a max–min DSF value δ. Because of constraint (9) and the objective function of *LP*2, we can guarantee that for any feasible DSF vector α', $\min\{\alpha'_i | 1 \leqslant i \leqslant N\} \leqslant \delta$. Next, we feed δ to *LP*3 as a parameter. Constraint (10) in *LP*3 guarantees that in the computed $\mathbf{r} = [r_1, r_2, \ldots, r_N]$ and its corresponding DFS vector $\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_N]$, we have $\min\{\alpha_i | 1 \leqslant i \leqslant N\} \geqslant \delta \geqslant \min\{\alpha'_i | 1 \leqslant i \leqslant N\}$. The objective of *LP*3 is to maximize the throughput. Therefore, solving *LP*2 and *LP*3(δ) together can provide a max–min fair maximum throughput solution.

The PASS problem can be formulated as a CP because it has the similar linear constraints as the MASS problem and the MMASS problem, and its objective is to maximize a *concave* utility function.

CP: PASS

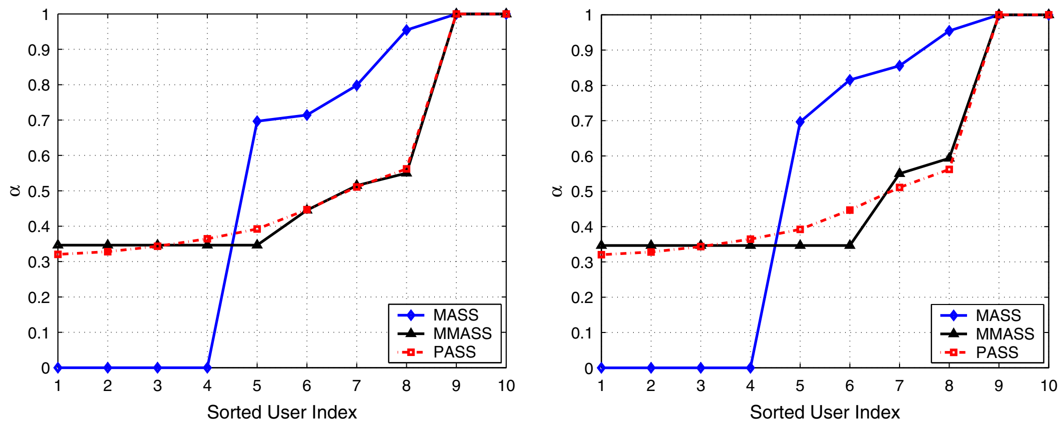
$$\max \quad \sum_{i=1}^{N} \log(\alpha_i)$$

subject to:

$$\alpha_i d_i \leqslant \sum_{j=1}^{C} \sum_{t:\Gamma_{ij}^t=1} p_t c_i^j, \quad 1 \leqslant i \leqslant N;$$
$$\sum_{t=1}^{T} p_t = 1; \tag{11}$$
$$p_t \geqslant 0, \quad 1 \leqslant t \leqslant T;$$
$$0 \leqslant \alpha_i \leqslant 1, \quad 1 \leqslant i \leqslant N.$$

Our two-step algorithms are summarized as follows: Step 1, construct the MCCG and apply the algorithms proposed in [14] or our Algorithm 1 to find all or a subset of transmission modes (which lead to optimal and suboptimal solutions, respectively); Step 2, solve LP1 for the MASS problem, solve LP2/LP3(δ) for the MMASS problem, or solve CP1 for the PASS problem. Note that the LP for MASS problem only include $(N+T)$ variables and $(2N+T+1)$ constraints, where N and T are the number of users and the number of transmission modes respectively. The LPs for the MMASS problem and the CP for the PASS problem have the similar complexities. So normally, they can all be efficiently solved by the existing algorithms.

Our two-step algorithms are suitable for cognitive radio wireless networks, in which the available channels to each user may vary frequently. Every time when an existing channel becomes no longer available to a user or a new channel becomes available to a user, we do not have to go through the whole two-step procedure to compute a completely new solution. We can simply eliminate those transmission modes including the user-channel pair which



(a) Scenario 10: heuristic algorithms with $q = 1$

(b) Scenario 11: heuristic algorithms with $q = 2$

Fig. 7. Physical model with $N = 10$, $C = 6$ and $d_i = [10.8, 25.2]$.

is no longer available from the current transmission mode set, or add one or more transmission modes including the newly available user-channel pairs to the existing set. Then we solve the corresponding LP or CP. In other words, we do not have to re-run Algorithm 1 to find a new set of transmission modes every time when the channel availability changes. In this way, we can obtain a new solution based on the updated channel availability in a time-efficient fashion. Of course, if substantial changes occur in the system after a certain period of time, in order to guarantee high performance, the whole two-step algorithm should be re-executed to compute a completely new solution.

5.3. Proposed algorithms for the physical model

If we assume that every node has the power control capability, the physical model should be used to address interference. In this case, we are unable to model the impact of interference using the MCCG because the one-to-one interference relationships among user-channel pairs are unavailable in the physical model. Therefore, the algorithms in [14] or our Algorithm 1 cannot be applied to find a set of transmission modes.

To our best knowledge, there is no algorithm in the literature which can identify all transmission modes under the physical model. However, a *good* subset of transmission modes can be identified efficiently by revising our Algorithm 1. Here, every time when we try to decide if a specific user-channel pair (i,j) (note that a node v in Algorithm 1 corresponds to a user-channel pair) can be selected to set S in Step 2 of Algorithm 1, instead of checking if it conflicts with another node in G_C which has already been selected to S, we verify the feasibility by solving $LP4(i,E_j)$. However, the user of a user-pair in the current S is incident to user i, we can conclude that (i,j) cannot be selected to S and no LP needs to be solved.

LP4(i,E_j):

$$\min \sum_{l\in E_j\bigcup\{i\}} P_l \tag{12}$$

subject to:

$$G^j_{T(l)R(l)}P_l - \beta \sum_{h\in E_j\bigcup\{i\}\setminus\{l\}} G^j_{T(h)R(l)}P_h - \beta N_0 \geqslant 0 \quad \forall l\in E_j\bigcup\{i\}; \tag{13}$$

$$0\leqslant P_l\leqslant P_{\max} \quad \forall l\in E_j\bigcup\{i\}. \tag{14}$$

In LP4(i,E_j), P_l is the variable which specifies the power level for user l on channel j. E_j denotes the current set of user-channels in S containing the same channel j. Again, $T(\cdot)$ and $R(\cdot)$ give the transmitter and the receiver of a given user respectively. If a feasible solution can be obtained by solving LP4(i,E_j), then we can conclude that user-channel pair (i,j) can be added to the current set S. This is because that in a feasible solution, the SINR constraint defined in the physical model is guaranteed to be satisfied for each user according to constraint (13) and the computed power level of each user is ensured to be in the range $[0,P_{\max}]$ according to constraint (14). Eventually, the solution given by LP4(h,E_j) can be used as the power assignment for the corresponding transmission mode. Even though we only need to obtain a feasible power assignment or to test if there exists a feasible solution, it is always good to minimize the total power consumption which is achieved by the objective function (12). In addition, the same weight function in Algorithm 1 can be used to determine which user-channel pair has the highest priority to be selected.

After identifying a set of transmission modes, we can then compute the rate allocation and scheduling solution by solving LP1, LP2/LP3(δ) or CP1.

6. Numerical results

In our simulation, we considered multihop cognitive radio wireless networks with stationary nodes randomly located in a region. We randomly chose N users (links) from a network in each run. For the protocol model, the transmission range and corresponding interference range of each user were set to 250 m and 500 m [16] for all channels, respectively. For the physical model, we set the thermal noise power $N_0=-90$ dB m, the SINR threshold $\beta=10$ dB and the maximum transmission power $P_{\max}=300$ mW [17]. The channel gain, G^j_{uv} was simply set to $1/d(u,v)^4$ for all channels, where $d(u,v)$ is the Euclidean distance between transmitter u and receiver v. All LPs were solved by using CPLEX 9.0 [13]. We implemented the barrier method introduced in [4] to solve all CPs by setting the related parameters as follows: $\epsilon=10^{-3}$, $\mu=120$ and $t^{(0)}=2$.

Intuitively, the following parameters may have significant impacts on system performance: the number of users (N), the total number of channels (C), the number of channels available to each user (C^i_A), the capacity of user-channel pair (c^j_i), the traffic demand on each user (d_i) and the tunable parameter in Algorithm 1 (q). We studied their impacts by setting these parameters to different values in different simulation scenarios. The DSF of each user (α_i), the throughput ($\sum_{i=1}^N r_i$) and the value of the utility function ($\sum_{i=1}^N \log(\alpha_i)$) were employed as performance metrics. In addition, the users were sorted in the non-descending order of their DSF values.

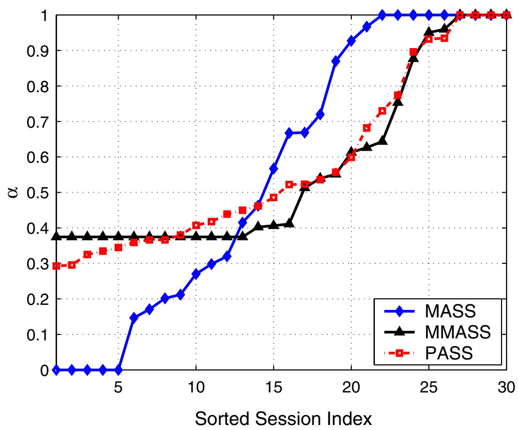


Fig. 8. Scenario 12: physical model with $N=30$, $C=12$ and $d_i=[10.8,25.2]$.

The simulation results are presented in Figs. 2–10. In all the figures, MASS, MMASS and PASS represent our algorithms for the MASS problem, the MMASS problem and the PASS problem respectively. We present network throughput and utility function results in Figs. 9 and 10 for all simulation scenarios. In the first nine scenarios, we evaluated our algorithms proposed for the protocol model. In scenario 1, we conducted simulations on the network given in Fig. 1a. In that example, $N = 5$, $C = 2$ and $C_A^1 = C_A^2 = 2$. Moreover, we set $c_i^j = 24$ Mbps, $\forall (i,j) \in \mathscr{A}$. The traffic demand for each user d_i was set to a random number uniformly distributed in [12,24] Mbps. The results are presented in Fig. 2. We actually run both our optimal algorithms, and heuristic algorithms by setting q to 1 and 2. However, we do not present results of heuristic algorithms because they are exactly the same as the optimal solutions. In scenarios 2–4, we performed simulation runs on a network with 10 nodes randomly distributed in a 500 m × 500 m area. 10 users were randomly selected. The other parameters were set as follows: $C = 6$, $C_A^i = 4, 1 \leqslant i \leqslant 10$. In addition, d_i was set to a random number uniformly distributed in $[0.3 * 24, 0.7 * 24]$ Mbps (i.e., $[7.2, 16.8]$) and c_i^j was randomly chosen from $\{24, 36\}$ Mbps. Note that these two rate values are typical data rates specified by IEEE802.11*a* [12]. We also executed both our optimal and heuristic algorithms by setting q to 1 and 2. The corresponding results are presented in Fig. 3. We conducted another set of simulation runs (scenarios 5–7) on the same network with the same settings except that the traffic demand for each user d_i was increased to a random number uniformly distributed in [12,24] Mbps. We presented the corresponding results in Fig. 4. In scenario 8, we tested our heuristic algorithms ($q = 2$) on a larger network with 30 nodes randomly distributed in a 1000 m × 1000 m area. Accordingly, 30 users were randomly selected. In addition, we had $C = 12$, $C_A^i = 8, 1 \leqslant i \leqslant 30$. The other settings are the same as those in scenario 2. The only difference between scenario 9 and scenario 8 is that the traffic demand for each user d_i was set to a random number uniformly distributed in [12,24] Mbps instead of $[7.2, 16.8]$ Mbps. The corresponding results are presented in Figs. 5,6. In the last three scenarios, scenarios 10–12, we evaluated the heuristic algorithms proposed for the physical model. In these scenarios, all user-channel pairs were assumed to have a capacity of 36 Mbps and the traffic demand for each user d_i was set to a random number uniformly distributed in $[0.3 * 36, 0.7 * 36]$ Mbps (i.e., $[10.8, 25.2]$). The results are presented in Figs. 7,8. The other settings of these three scenarios are the same as those in scenarios 3, 4 and 8 respectively.

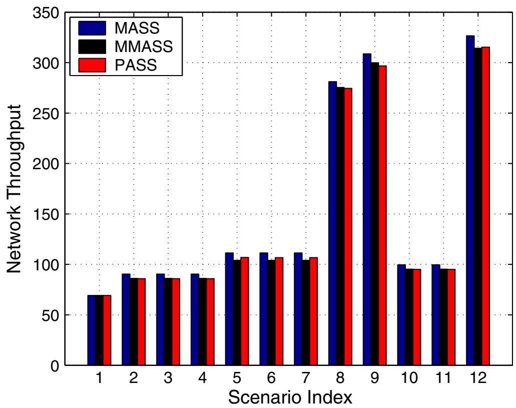


Fig. 9. Network throughput in different scenarios.

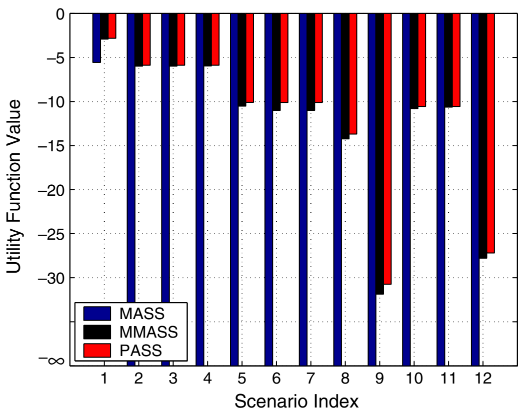


Fig. 10. Utility function values in different scenarios.

From Figs. 3, 4 and 9, we can see that the performance achieved by our heuristic algorithms (with $q = 2$) is almost the same as that of the optimal solutions with regards to both throughput and fairness. In addition, adding more transmission modes for consideration by increasing parameter q from 1 to 2 does not provide a noticeable throughput improvement no matter which algorithm is used.

As expected, we observe that the MASS algorithms perform best in terms of throughput but suffer from a severe unfairness on rate allocation among users in all simulation scenarios. For example, in scenario 5 (Fig. 4a), the traffic demands of about half of users are not satisfied at all ($\alpha = 0$). However, all the other users obtain very high DSF values. Fig. 10 shows that the values of utility function given by the MASS algorithm are always very small, which also illustrates its unfairness on rate allocation. The MMASS algorithms give the max–min DSF values which can be clearly observed from the results of all scenarios. Compared to the MASS algorithms, the PASS algorithms offer very close throughput in all scenarios (Fig. 9). The average throughput given by the PASS algorithms is 96.3% of the maximum achievable throughput. Moreover, they always give the best utility function values (Fig. 10), which indicates their efficiency in fairness.

7. Conclusions

In this paper, we have studied the joint spectrum allocation and scheduling in cognitive radio wireless networks. Specifically, under the protocol interference model, we proposed a novel Multi-Channel Contention Graph (MCCG) to characterize the impact of interference. We have formally defined the MASS problem, the MMASS problem, and the PASS problem. For each problem, we presented an optimal algorithm and a fast heuristic algorithm based on the MCCG. In addition, we proposed fast and effective heuristics to solve those problems under the physical interference model. Our numerical results have shown that the performance given by our heuristic algorithms is very close to that of the optimal solutions. Furthermore, a good tradeoff between throughput and fairness can be achieved by our PASS algorithms.

References

[1] M. Alicherry, R. Bhatia, L. Li, Joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks, in: Proceedings of ACM MobiCom'2005, pp. 58–72.
[2] I.F. Akyildiz, W-Y Lee, M.C. Vuran, S. Mohanty, NeXt generation/dynamic spectrum access/cognitive radio wireless networks: a survey, Elsevier Journal of Computer Networks 50 (13) (2007) 2127–2159.
[3] M.S. Bazaraa, J.J. Jarvis, H.D. Sherali, Linear Programming and Network Flows, third ed., John Wiley & Sons, 2005.
[4] S. Boyd, L. Vandenberghe, Convex Optimization, Cambridge University Press, 2004.
[5] V. Brik, E. Rozner, S. Banarjee and P. Bahl, DSAP: a protocol for coordinated spectrum access, Proceedings of IEEE DySPAN'2005, pp. 611–614.
[6] L. Cao and H. Zheng, Distributed spectrum allocation via local bargaining, Proceedings of IEEE Secon'2005, pp. 475–486.
[7] P. Gupta, P.R. Kumar, The capacity of wireless networks, IEEE Transactions on Information Theory 46 (2) (2000) 388–404.
[8] M. Halldorsson, H.C. Lau, Low-degree graph partitioning via local search with applications to constraint satisfaction, max cut, and 3-coloring, Journal of Graph Algorithms and Applications 1 (3) (1997) 1–13.
[9] Y.T. Hou, Y. Shi, H.D. Sherali, Rate allocation in wireless sensor networks with network lifetime requirement, in: Proceedings of ACM MobiHoc'2004, pp. 67–77.
[10] Y.T. Hou, Y. Shi, H.D. Sherali, Optimal spectrum sharing for multi-hop software defined radio networks, in: Proceedings of IEEE Infocom'2007, pp. 1–9.
[11] J. Huang, R.A. Berry, M.L. Honig, Spectrum sharing with distributed interference compensation, in: Proceedings of IEEE DySPAN'2005, pp. 88–93.
[12] IEEE 802.11a Working Group, Wireless LAN MAC and PHY Specifications – Amendment 1: High-Speed Physical Layer in the 5 GHz Band, IEEE Standard, 1999.
[13] ILOG Software Inc., CPLEX 9.0. <http://www.ilog.com>.
[14] D. Johnson, M. Yannakakis, C.H. Papadimitriou, On generating all maximal independent sets, Information Processing Letters 27 (3) (1988) 119–123.
[15] T. Nandagopal, T.E. Kim, X. Gao, V. Bharghavan, Achieving MAC layer fairness in wireless packet networks, in: Proceedings of ACM MobiCom'2000, pp. 87–98.
[16] A. Raniwala, T. Chiueh, Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network, in: Proceedings of IEEE Infocom'2005, pp. 2223–2234.
[17] J. Tang, G. Xue, C. Chandler, W. Zhang, Link scheduling with power control for throughput enhancement in multihop wireless networks, IEEE Transactions on Vehicular Technology (TVT) 5 (3) (2006) 733–742.
[18] J. Tang, G. Xue, W. Zhang, Cross-layer design for end-to-end throughput and fairness enhancement in multi-channel wireless mesh networks, IEEE Transactions on Wireless Communications 6 (10) (2007) 3482–3486.
[19] J. Tang, G. Xue, W. Zhang, Cross-layer optimization for end-to-end rate allocation in multi-radio wireless mesh networks, ACM/Kluwer Journal of Wireless Networks, in press.
[20] J. Wang, Y. Fang, D. Wu, A power-saving multi-radio multi-channel MAC protocol for wireless local area networks, in: Proceedings of IEEE Infocom'2006.
[21] Q. Wang, H. Zheng, Route and spectrum selection in dynamic spectrum networks, in: Proceedings of IEEE CNCC'2006.
[22] Y. Xi, E.M. Yeh, Distributed algorithms for spectrum allocation, power control, routing, and congestion control in wireless networks, in: Proceedings of ACM MobiHoc'2007, pp. 180–189.
[23] C. Xin, B. Xie, C-C. Shen, A novel layered graph model for topology formation and routing in dynamic spectrum access networks, in: Proceedings of IEEE DySPAN'2005, pp. 308–317.
[24] Y. Yuan, P. Bahl, R. Chandra, T. Moscibroda and Y. Wu, Allocating dynamic time-spectrum blocks in cognitive radio networks, in: Proceedings of ACM MobiHoc'2007, pp. 130–139.
[25] Q. Zhao, L. Tong, A. Swami, Decentralized cognitive MAC for dynamic spectrum access, in: Proceedings of IEEE DySPAN'2005, pp. 224–232.
[26] H. Zheng and C. Peng, Collaboration and fairness in opportunistic spectrum access, in: Proceedings of IEEE ICC'2005, pp. 3132–3136.

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