

**CS 436 Artificial  
Intelligence  
AIMA Chapter 7 – Logical  
Agents**

**Lecturer: Neal Richter**

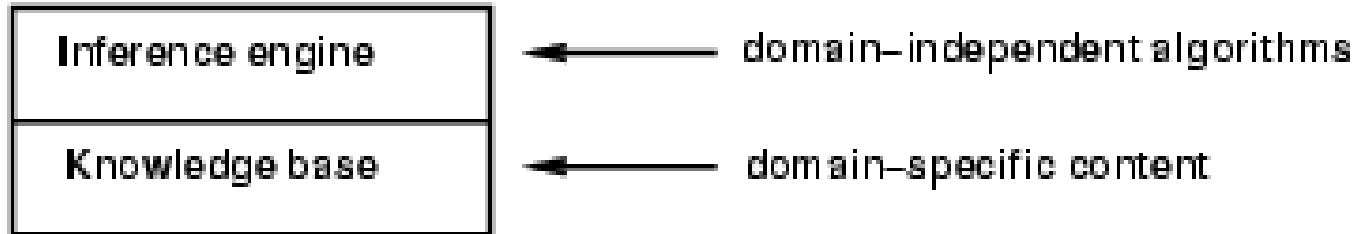
Montana State University -- Fall 2006



# Outline

- Knowledge-based agents
- Wumpus world
- Logic in general - models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - forward chaining
  - backward chaining
  - resolution

# Knowledge bases



- Knowledge base = set of **sentences** in a **formal** language
- **Declarative** approach to building an agent (or other system):
  - Tell it what it needs to know
- Then it can **Ask** itself what to do - answers should follow from the KB
- Agents can be viewed at the **knowledge level**  
i.e., what they know, regardless of how implemented
- Or at the **implementation level**
  - i.e., data structures in KB and algorithms that manipulate them

# A simple knowledge-based agent

```
function KB-AGENT(percept) returns an action
  static: KB, a knowledge base
         t, a counter, initially 0, indicating time

  TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
  action ← ASK(KB, MAKE-ACTION-QUERY(t))
  TELL(KB, MAKE-ACTION-SENTENCE(action, t))
  t ← t + 1
  return action
```

- The agent must be able to:
  - Represent states, actions, etc.
  - Incorporate new percepts
  - Update internal representations of the world
  - Deduce hidden properties of the world
  - Deduce appropriate actions

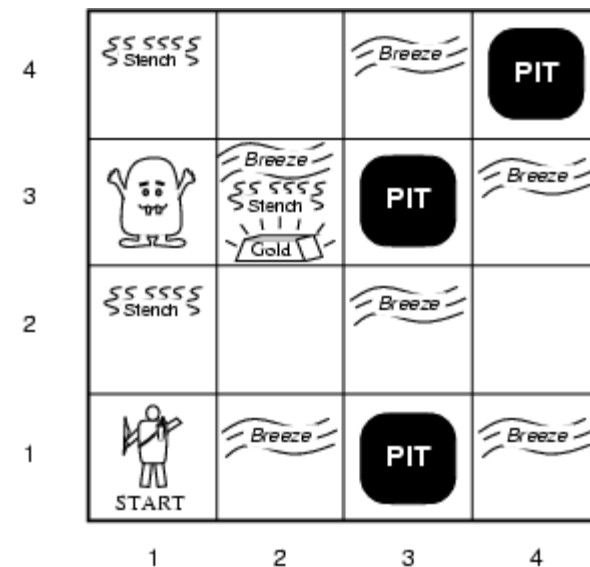
# Wumpus World PEAS description

## ■ Performance measure

- gold +1000, death -1000
- -1 per step, -10 for using the arrow

## ■ Environment

- Squares adjacent to wumpus are smelly
- Squares adjacent to pit are breezy
- Glitter iff gold is in the same square
- Shooting kills wumpus if you are facing it
- Shooting uses up the only arrow
- Grabbing picks up gold if in same square
- Releasing drops the gold in same square



## ■ Sensors: Stench, Breeze, Glitter, Bump, Scream

## ■ Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot



# Wumpus world characterization

- Fully Observable No – only local perception
- Deterministic Yes – outcomes exactly specified
- Episodic No – sequential at the level of actions
- Static Yes – Wumpus and Pits do not move
- Discrete Yes
- Single-agent? Yes – Wumpus is essentially a natural feature

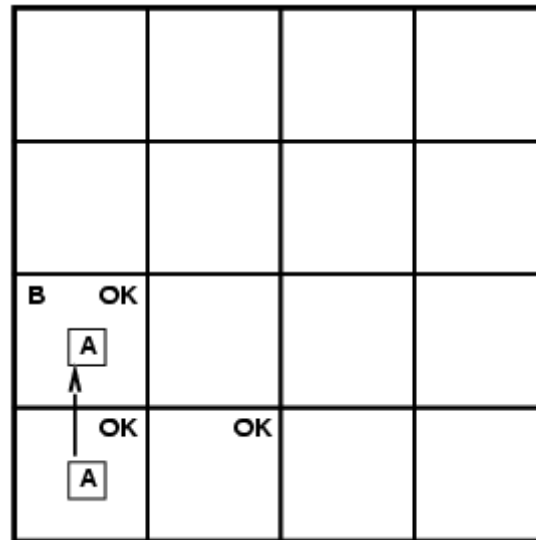
# Exploring a wumpus world

OK			
OK A	OK		

**Initial Percept = [None, None, None, None, None]**

**Percept Code = [Stench?, Breeze?, Glitter?, Bump?, Scream?]**

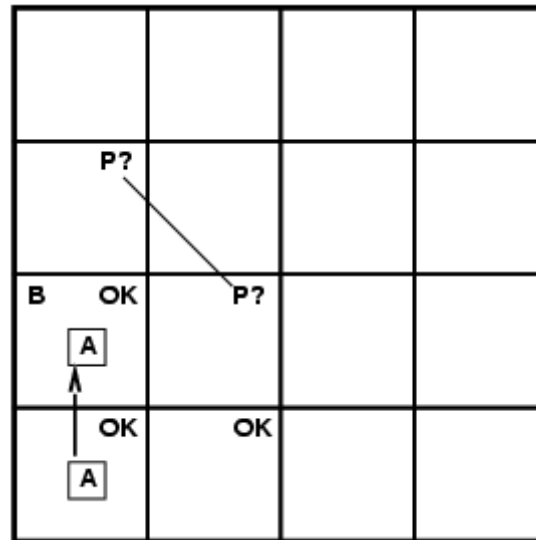
# Exploring a wumpus world



**Initial Percept = [None, Breeze, None, None, None]**

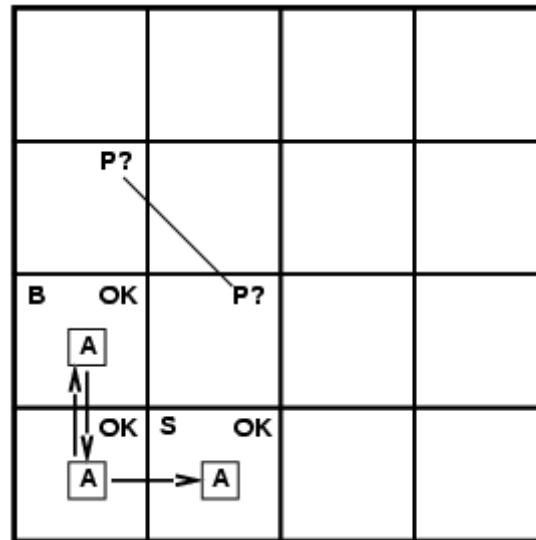
**Percept Code = [Stench?, Breeze?, Glitter?, Bump?, Scream?]**

# Exploring a wumpus world



**Deductions based upon rules**

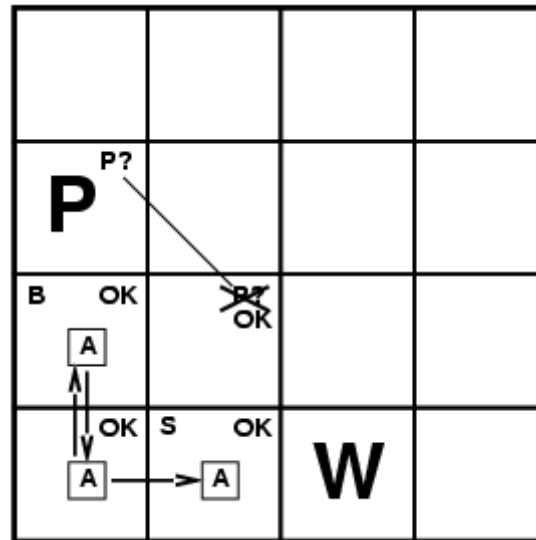
# Exploring a wumpus world



**Initial Percept = [Stench , None, None, None, None]**

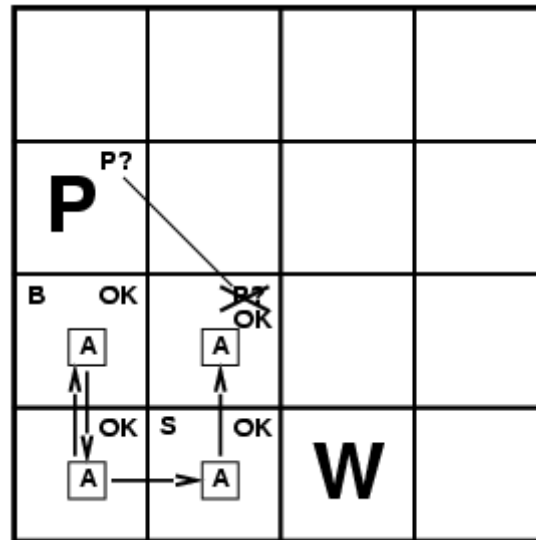
**Percept Code = [Stench?, Breeze?, Glitter?, Bump?, Scream?]**

# Exploring a wumpus world



Deductions based upon rules

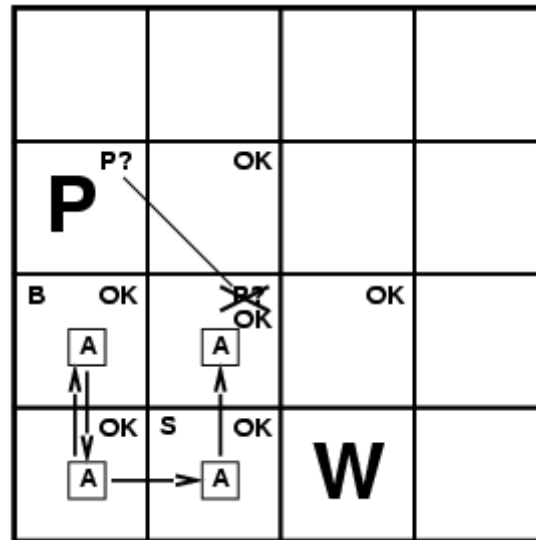
# Exploring a wumpus world



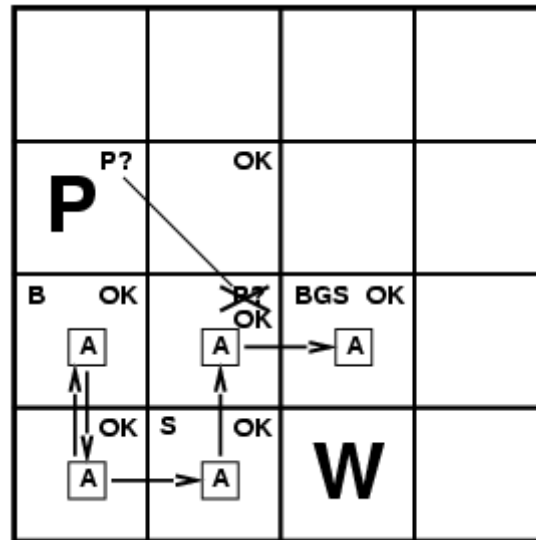
Initial Percept = [None, None, None, None, None]

Percept Code = [Stench?, Breeze?, Glitter?, Bump?, Scream?]

# Exploring a wumpus world



# Exploring a wumpus world



Initial Percept = [Stench, Breeze, Glitter, None, None]

Percept Code = [Stench?, Breeze?, Glitter?, Bump?, Scream?]



# Logic in general

- **Logics** are formal languages for representing information such that conclusions can be drawn
- **Syntax** defines the sentences in the language
- **Semantics** define the "meaning" of sentences;
  - i.e., define **truth** of a sentence in a world
- E.g., the language of arithmetic
  - $x+2 \geq y$  is a sentence;  $x^2+y > \{ \}$  is not a sentence
  - $x+2 \geq y$  is true iff the number  $x+2$  is no less than the number  $y$
  - $x+2 \geq y$  is true in a world where  $x = 7, y = 1$
  - $x+2 \geq y$  is false in a world where  $x = 0, y = 6$

# Entailment

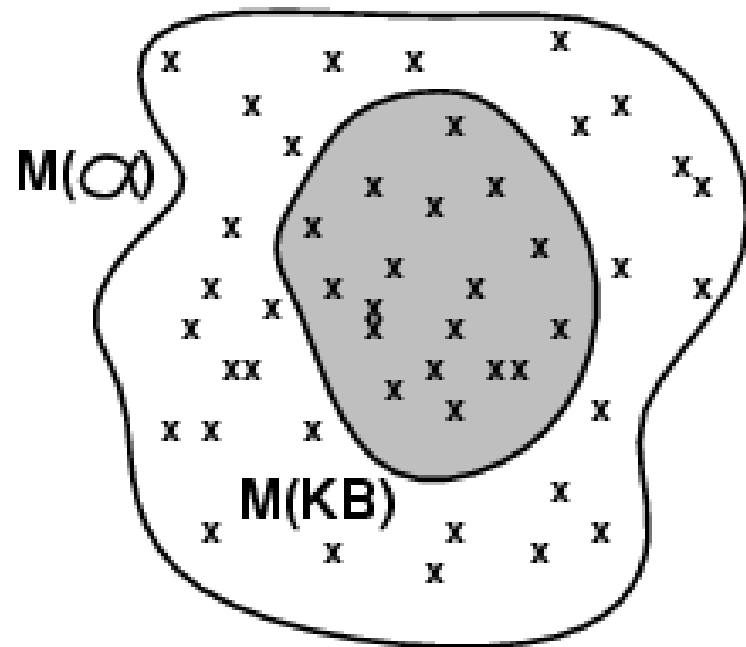
- Entailment means that one thing **follows from** another:

$$KB \models \alpha$$

- The truth of  $\alpha$  is contained in the truth of  $KB$
- Knowledge base  $KB$  entails sentence  $\alpha$  if and only if  $\alpha$  is true in all worlds where  $KB$  is true
  - E.g., the  $KB$  containing “the Giants won” and “the Reds won” entails “Either the Giants won or the Reds won”
  - E.g.,  $x+y = 4$  entails  $4 = x+y$
  - Entailment is a relationship between sentences (i.e., **syntax**) that is based on **semantics**

# Models

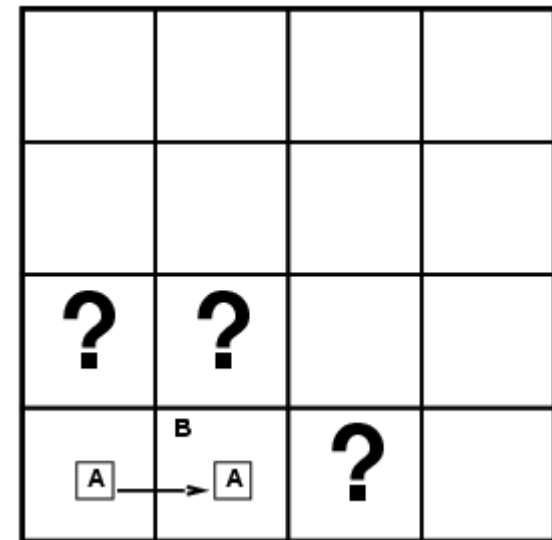
- Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated
- We say  $m$  is a **model** of a sentence  $\alpha$
- $M(\alpha)$  is the set of all models of  $\alpha$
- Then  $KB \models \alpha$  iff  $M(KB) \subseteq M(\alpha)$ 
  - E.g.  $KB =$  Giants won and Reds won  $\alpha =$  Giants won



# Entailment in the wumpus world

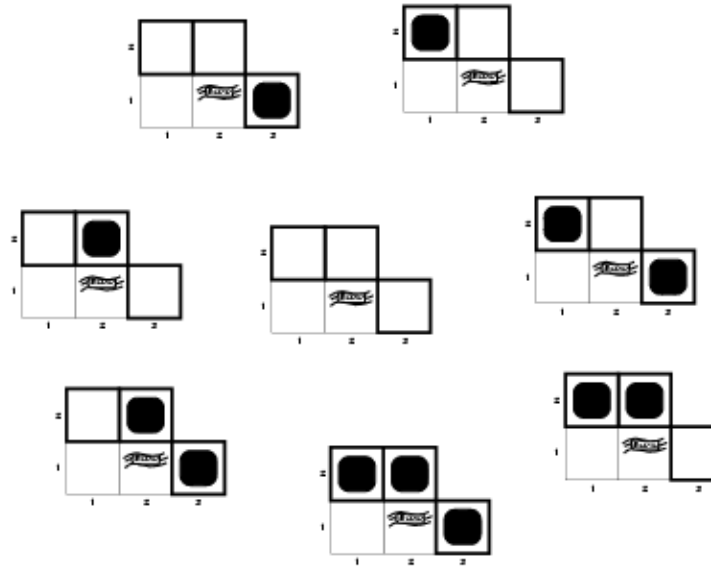
Situation after detecting  
nothing in [1,1], moving  
right, breeze in [2,1]

Consider possible models for  
*KB* assuming only pits



3 Boolean choices  $\Rightarrow$  8  
possible models

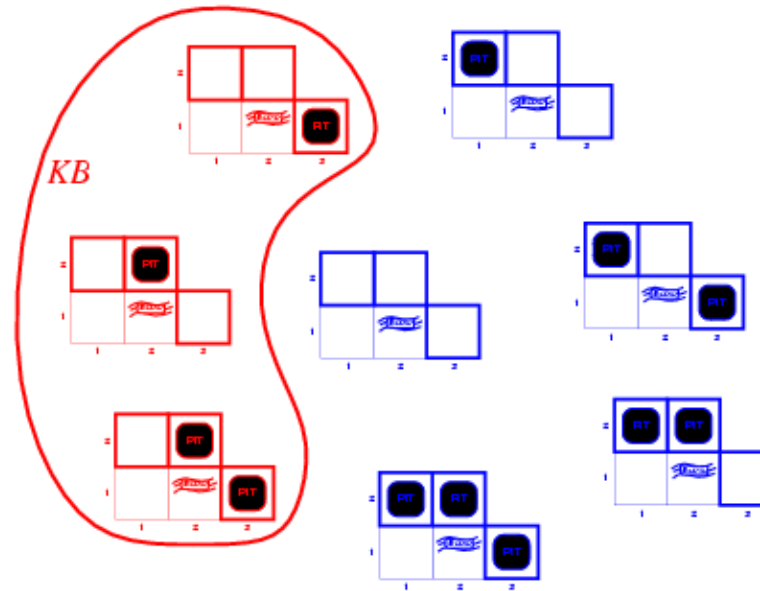
# Wumpus models



Complete 'Model Space' of the 'Pit' variable in play.

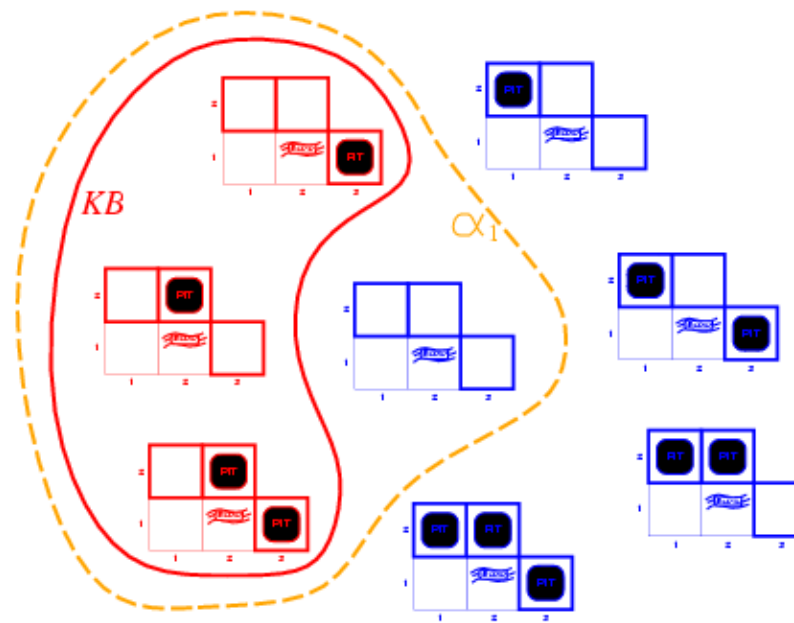
Which ones are possible given our Percepts?

# Wumpus models



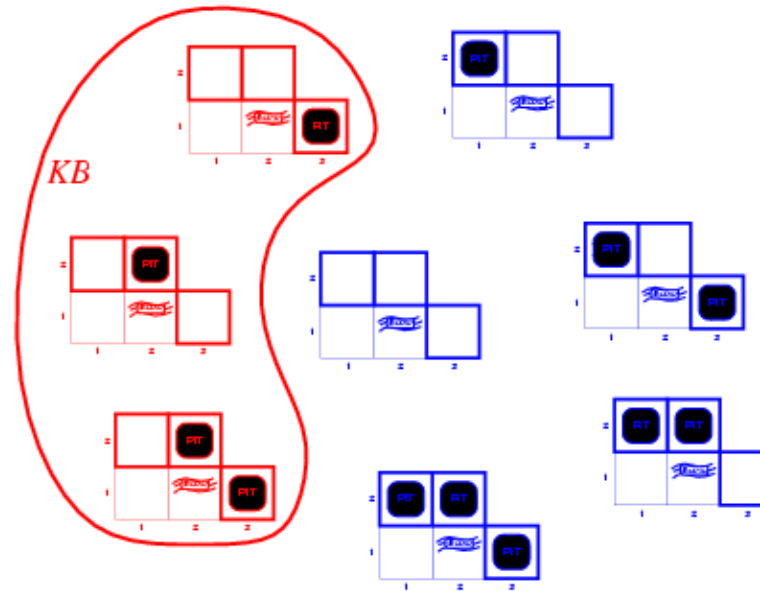
- $KB = \text{wumpus-world rules} + \text{observations}$

# Wumpus models



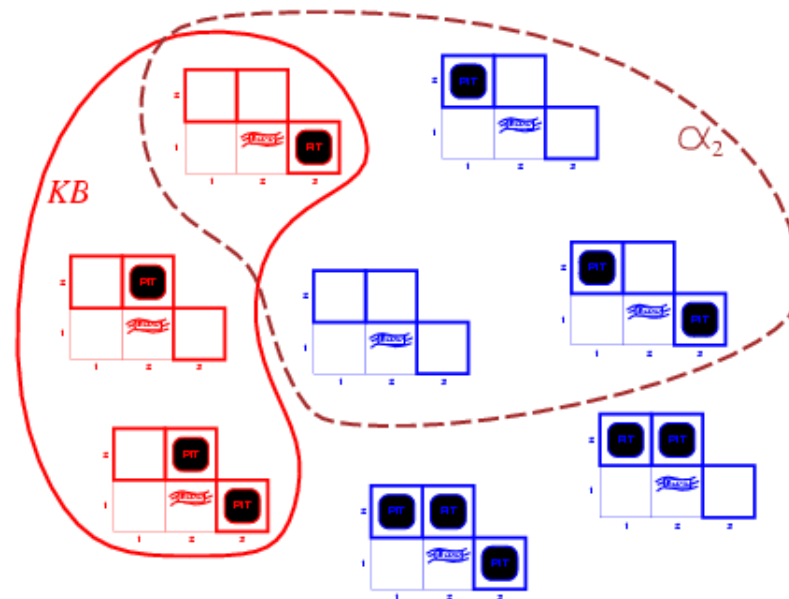
- $KB$  = wumpus-world rules + observations
- $\alpha_1 = "[1,2]$  is safe",  $KB \models \alpha_1$ , proved by model checking
- Why? - There is no breeze in  $[1,1]$

# Wumpus models



- $KB = \text{wumpus-world rules} + \text{observations}$

# Wumpus models



- $KB$  = wumpus-world rules + observations
- $\alpha_2$  = "[2,2] is safe",  $KB \not\models \alpha_2$
- $\alpha_2$  Contradicts our model



# Inference

- $KB \vdash_i \alpha$  = sentence  $\alpha$  can be derived from  $KB$  by inference procedure  $i$
- **Soundness**:  $i$  is sound if whenever  $KB \vdash_i \alpha$ , it is also true that  $KB \models \alpha$
- **Completeness**:  $i$  is complete if whenever  $KB \models \alpha$ , it is also true that  $KB \vdash_i \alpha$
- Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.
- That is, the procedure will answer any question whose answer follows from what is known by the  $KB$ .



# Propositional logic: Syntax

- Propositional logic is the simplest logic – illustrates basic ideas
- The proposition symbols  $P_1, P_2$  etc are sentences
  - If  $S$  is a sentence,  $\neg S$  is a sentence (**negation**) □
  - If  $S_1$  and  $S_2$  are sentences,  $S_1 \wedge S_2$  is a sentence (**conjunction**)
  - If  $S_1$  and  $S_2$  are sentences,  $S_1 \vee S_2$  is a sentence (**disjunction**)
  - If  $S_1$  and  $S_2$  are sentences,  $S_1 \Rightarrow S_2$  is a sentence (**implication**)
  - If  $S_1$  and  $S_2$  are sentences,  $S_1 \Leftrightarrow S_2$  is a sentence (**biconditional**)

# Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.  $P_{1,2}$        $P_{2,2}$        $P_{3,1}$   
false            true            false

With these symbols, 8 possible models, can be enumerated automatically.

Rules for evaluating truth with respect to a model  $m$ :

$\neg S$	is true iff	$S$ is false
$S_1 \wedge S_2$	is true iff	$S_1$ is true <b>and</b> $S_2$ is true
$S_1 \vee S_2$	is true iff	$S_1$ is true <b>or</b> $S_2$ is true
$S_1 \Rightarrow S_2$	is true iff	$S_1$ is false <b>or</b> $S_2$ is true
i.e.,	is false iff	$S_1$ is true <b>and</b> $S_2$ is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true <b>and</b> $S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \text{true} \wedge (\text{true} \vee \text{false}) = \text{true} \wedge \text{true} = \text{true}$$



# Truth tables for connectives

$P$	$Q$	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>



# Wumpus world sentences

Let  $P_{i,j}$  be true if there is a pit in  $[i, j]$ .

Let  $B_{i,j}$  be true if there is a breeze in  $[i, j]$

$$\neg P_{1,1}$$

$$\neg B_{1,1}$$

$$B_{2,1}$$

- "Pits cause breezes in adjacent squares"

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

# Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	$KB$	$\alpha_1$
<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>
<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>true</i>
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<u><i>true</i></u>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<u><i>true</i></u>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<u><i>true</i></u>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>

- $KB$  itself is true for only three rows.. When  $R1$  through  $R5$  are true.

# Inference by enumeration

- Depth-first enumeration of all models is sound and complete

```
function TT-ENTAILS?(KB,  $\alpha$ ) returns true or false
  symbols ← a list of the proposition symbols in KB and  $\alpha$ 
  return TT-CHECK-ALL(KB,  $\alpha$ , symbols, [])

function TT-CHECK-ALL(KB,  $\alpha$ , symbols, model) returns true or false
  if EMPTY?(symbols) then
    if PL-TRUE?(KB, model) then return PL-TRUE?( $\alpha$ , model)
    else return true
  else do
    P ← FIRST(symbols); rest ← REST(symbols)
    return TT-CHECK-ALL(KB,  $\alpha$ , rest, EXTEND(P, true, model)) and
      TT-CHECK-ALL(KB,  $\alpha$ , rest, EXTEND(P, false, model))
```

- For  $n$  symbols, time complexity is  $O(2^n)$ , space complexity is  $O(n)$

# Logical equivalence

- Two sentences are **logically equivalent** iff true in same models:  $\alpha \equiv \beta$  iff  $\alpha \models \beta$  and  $\beta \models \alpha$

$$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge$$

$$(\alpha \vee \beta) \equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee$$

$$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge$$

$$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee$$

$$\neg(\neg\alpha) \equiv \alpha \quad \text{double-negation elimination}$$

$$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition}$$

$$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta) \quad \text{implication elimination}$$

$$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination}$$

$$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan}$$

$$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan}$$

$$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee$$

$$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge$$



# Validity and satisfiability

A sentence is **valid** if it is true in **all** models,  
e.g., *True*,  $A \vee \neg A$ ,  $A \Rightarrow A$ ,  $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the **Deduction Theorem**:  
 $KB \models \alpha$  if and only if  $(KB \Rightarrow \alpha)$  is valid

A sentence is **satisfiable** if it is true in **some** model  
e.g.,  $(R1 \wedge R2 \wedge R3 \wedge R4 \wedge R5)$

A sentence is **unsatisfiable** if it is true in **no** models  
e.g.,  $A \wedge \neg A$

Satisfiability is connected to inference via the following:  
 $KB \models \alpha$  if and only if  $(KB \wedge \neg \alpha)$  is unsatisfiable



# Proof methods

- Proof methods divide into (roughly) two kinds:
  - Application of inference rules
    - Legitimate (sound) generation of new sentences from old
    - Proof = a sequence of inference rule applications
      - Can use inference rules as operators in a standard search algorithm
    - Typically require transformation of sentences into a normal form
  - Model checking
    - truth table enumeration (always exponential in  $n$ )
    - improved backtracking, e.g., Davis--Putnam-Logemann-Loveland (DPLL)
    - heuristic search in model space (sound but incomplete)
      - e.g., min-conflicts-like hill-climbing algorithms

# Resolution

## Conjunctive Normal Form (CNF)

conjunction of disjunctions of literals .. eg clauses

E.g.,  $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

### Resolution inference rule (for CNF)

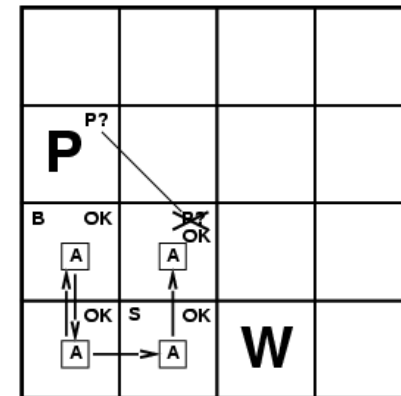
$$\frac{\begin{array}{c} \ell_i \vee \dots \vee \ell_k, \\ m_1 \vee \dots \vee m_n \end{array}}{\ell_i \vee \dots \vee \ell_{i-1} \vee \ell_{i+1} \vee \dots \vee \ell_k \vee m_1 \vee \dots \vee m_{j-1} \vee m_{j+1} \vee \dots \vee m_n}$$

□

where  $\ell_i$  and  $m_j$  are complementary literals.

$$\text{E.g., } \frac{P_{1,3} \vee P_{2,2}, \quad \neg P_{2,2}}{P_{1,3}}$$

### Resolution is sound and complete for propositional logic



# Conversion to CNF

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

1. Eliminate  $\Leftrightarrow$ , replacing  $\alpha \Leftrightarrow \beta$  with  $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$ .

$$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$$

2. Eliminate  $\Rightarrow$ , replacing  $\alpha \Rightarrow \beta$  with  $\neg\alpha \vee \beta$ .

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$

3. Move  $\neg$  inwards using de Morgan's rules and double-negation:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \vee \neg P_{2,1}) \vee B_{1,1})$$

4. Apply distributivity law ( $\wedge$  over  $\vee$ ) and flatten:

$$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

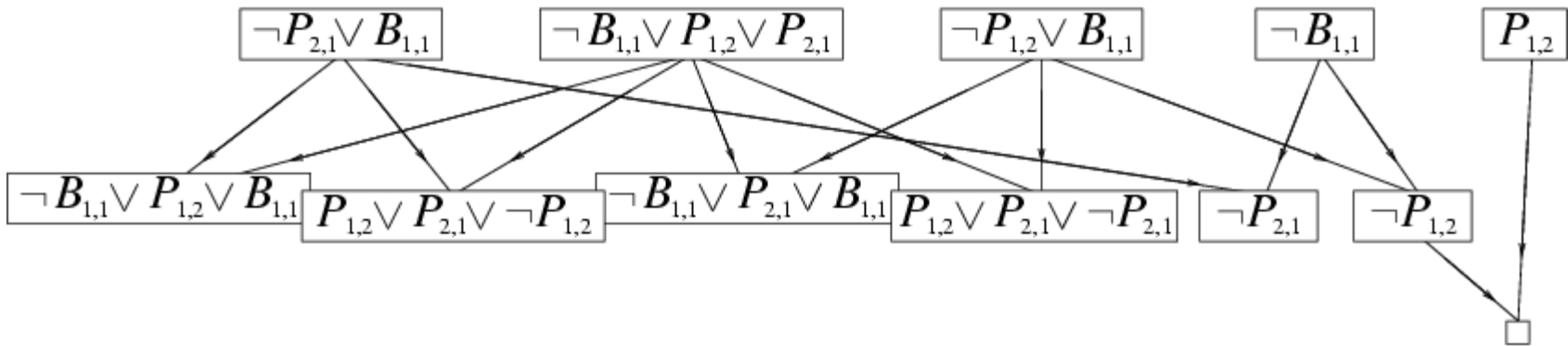
# Resolution algorithm

- Proof by contradiction, i.e., show  $KB \wedge \neg\alpha$  unsatisfiable

```
function PL-RESOLUTION( $KB, \alpha$ ) returns true or false
   $clauses \leftarrow$  the set of clauses in the CNF representation of  $KB \wedge \neg\alpha$ 
   $new \leftarrow \{ \}$ 
  loop do
    for each  $C_i, C_j$  in  $clauses$  do
       $resolvents \leftarrow$  PL-RESOLVE( $C_i, C_j$ )
      if  $resolvents$  contains the empty clause then return true
       $new \leftarrow new \cup resolvents$ 
    if  $new \subseteq clauses$  then return false
   $clauses \leftarrow clauses \cup new$ 
```

# Resolution example

- $KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$ 
 $\alpha = \neg P_{1,2}$



# Forward and backward chaining

- Horn Form (restricted)

KB = conjunction of Horn clauses

- Horn clause =

- proposition symbol; or
- (conjunction of symbols)  $\Rightarrow$  symbol

- E.g.,  $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

- Modus Ponens (for Horn Form): complete for Horn KBs

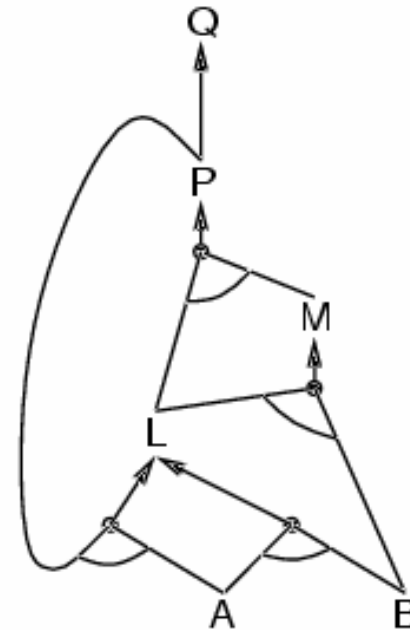
$$\frac{\alpha_1, \dots, \alpha_n, \quad \alpha_1 \wedge \dots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

- Can be used with forward chaining or backward chaining.
- These algorithms are very natural and run in linear time

# Forward chaining

- Idea: fire any rule whose premises are satisfied in the *KB*,
  - add its conclusion to the *KB*, until query is found

$P \Rightarrow Q$   
 $L \wedge M \Rightarrow P$   
 $B \wedge L \Rightarrow M$   
 $A \wedge P \Rightarrow L$   
 $A \wedge B \Rightarrow L$   
 $A$   
 $B$



# Forward chaining algorithm

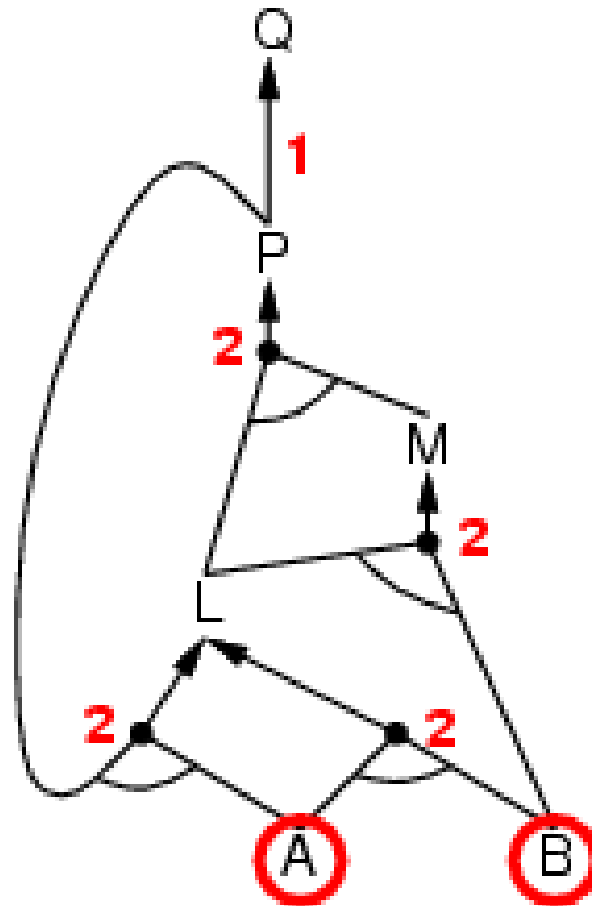
```
function PL-FC-ENTAILS?(KB, q) returns true or false
  local variables: count, a table, indexed by clause, initially the number of premises
                  inferred, a table, indexed by symbol, each entry initially false
                  agenda, a list of symbols, initially the symbols known to be true

  while agenda is not empty do
    p ← POP(agenda)
    unless inferred[p] do
      inferred[p] ← true
      for each Horn clause c in whose premise p appears do
        decrement count[c]
        if count[c] = 0 then do
          if HEAD[c] = q then return true
          PUSH(HEAD[c], agenda)

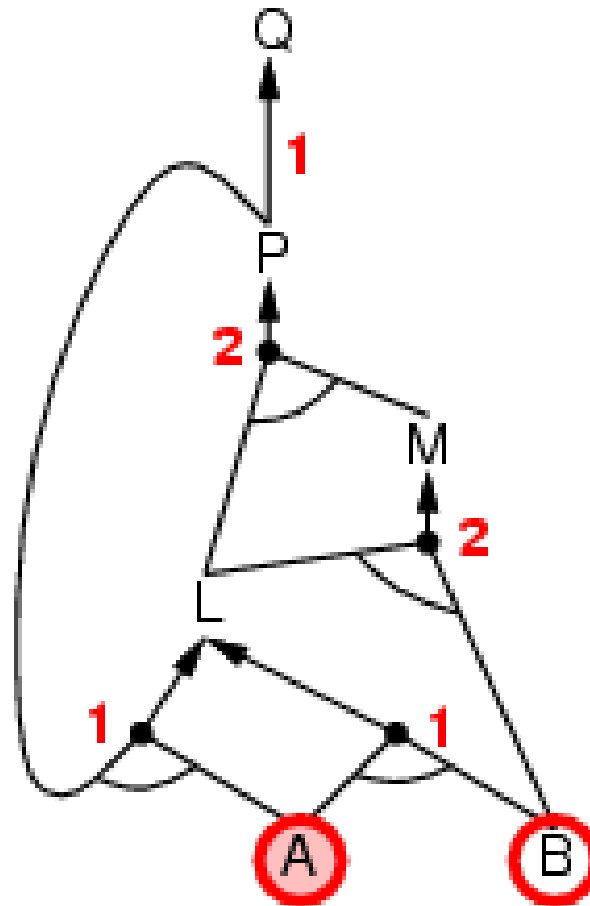
  return false
```

- Forward chaining is sound and complete for Horn KB

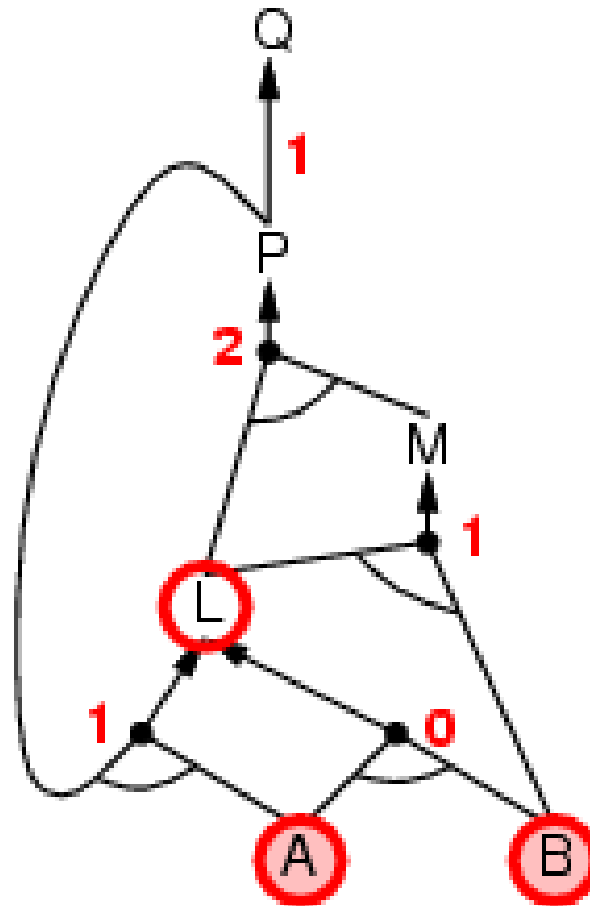
# Forward chaining example



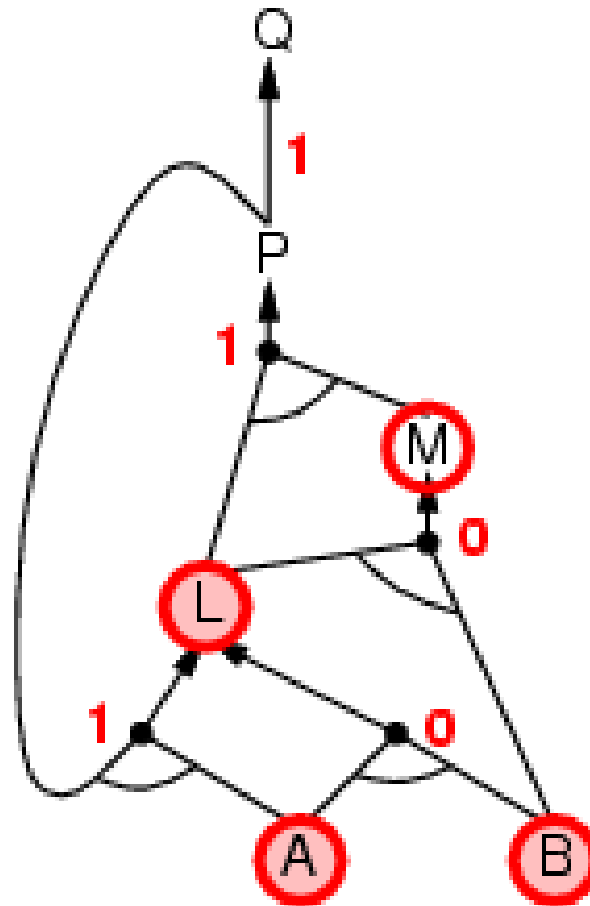
# Forward chaining example



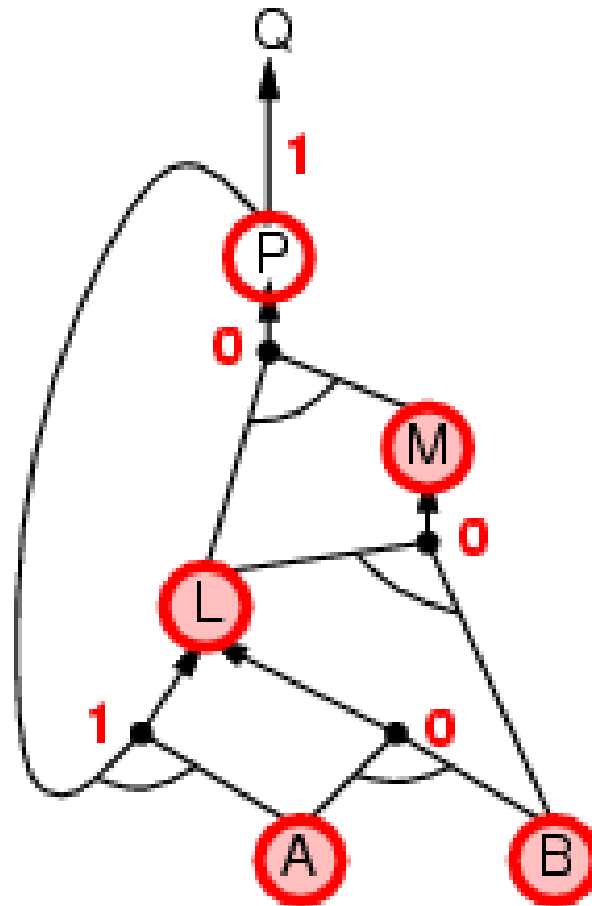
# Forward chaining example



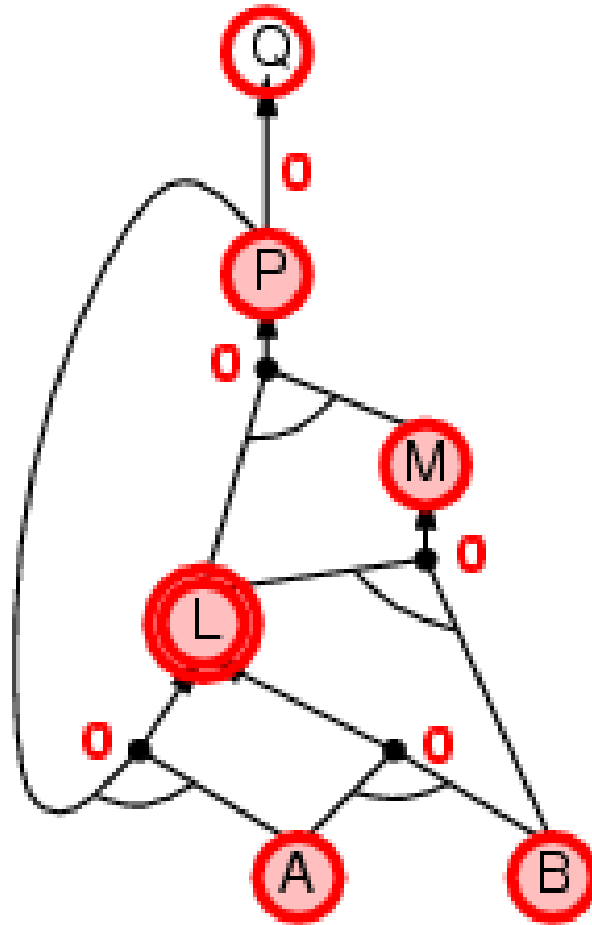
# Forward chaining example



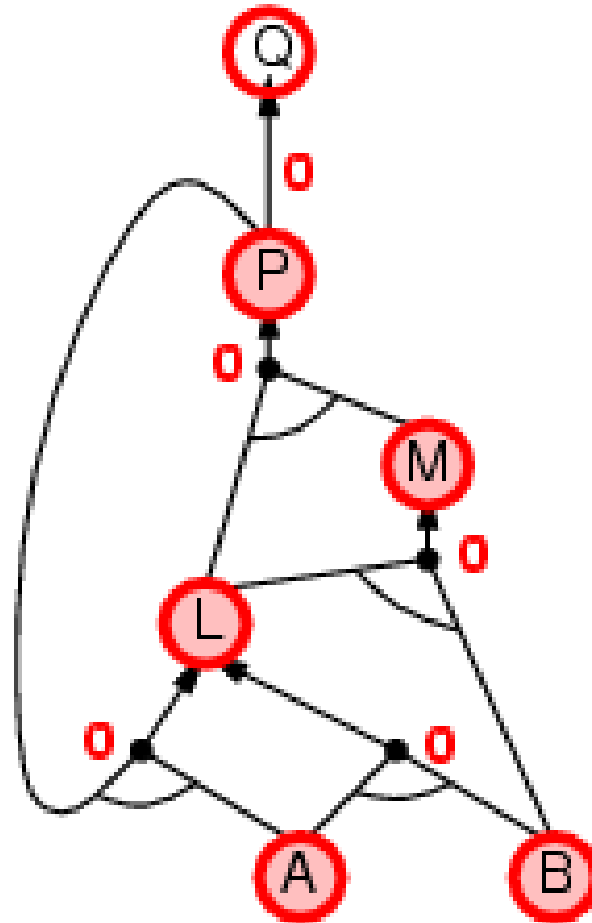
# Forward chaining example



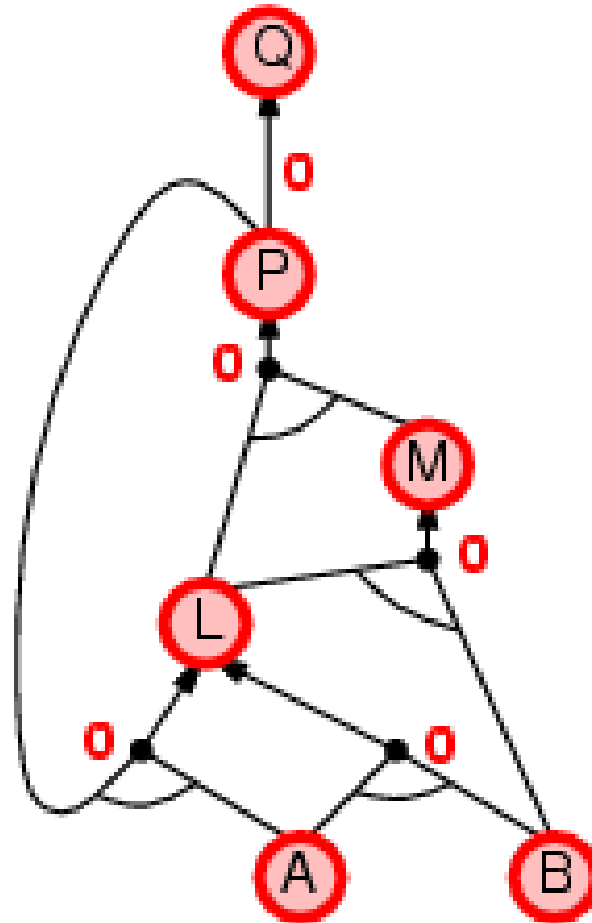
# Forward chaining example



# Forward chaining example



# Forward chaining example





# Proof of completeness

- FC derives every atomic sentence that is entailed by  $KB$ 
  1. FC reaches a **fixed point** where no new atomic sentences are derived
  2. Consider the final state as a model  $m$ , assigning true/false to symbols
  3. Every clause in the original  $KB$  is true in  $m$   
 $a_1 \wedge \dots \wedge a_k \Rightarrow b$
  4. Hence  $m$  is a model of  $KB$
  5. If  $KB \models q$ ,  $q$  is true in **every** model of  $KB$ , including  $m$



# Backward chaining

Idea: work backwards from the query  $q$ :

to prove  $q$  by BC,

check if  $q$  is known already, or

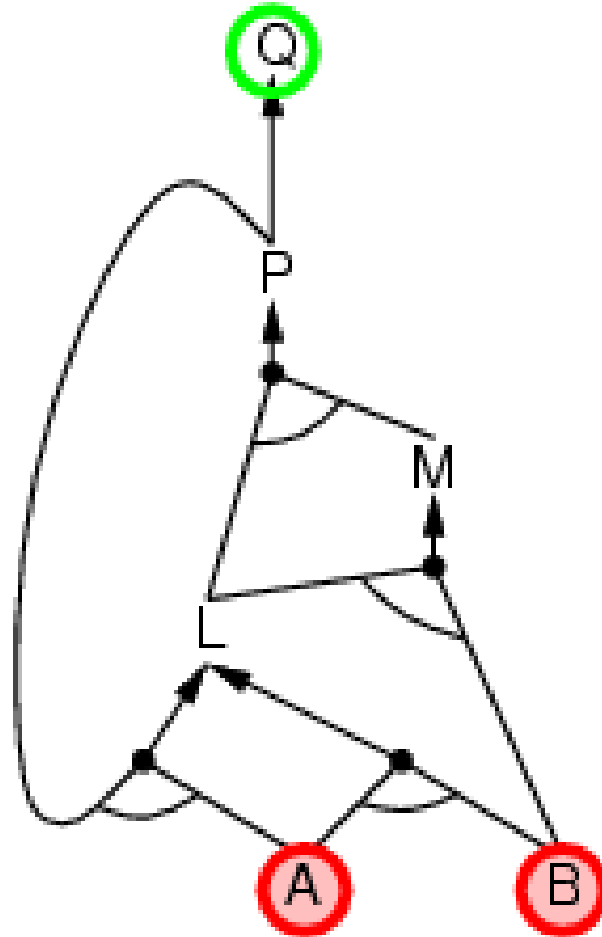
prove by BC all premises of some rule concluding  $q$

Avoid loops: check if new subgoal is already on the goal stack

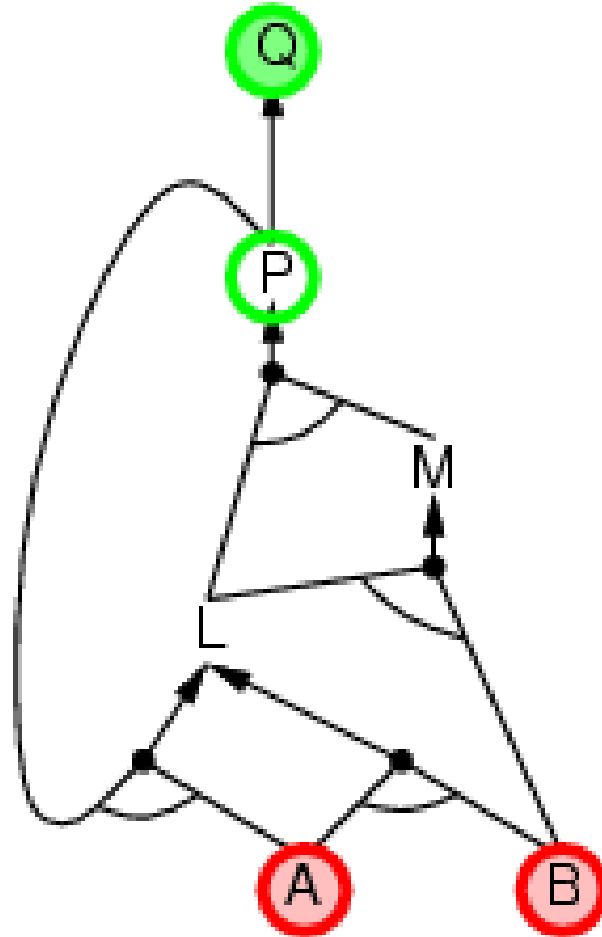
Avoid repeated work: check if new subgoal

1. has already been proved true, or
2. has already failed

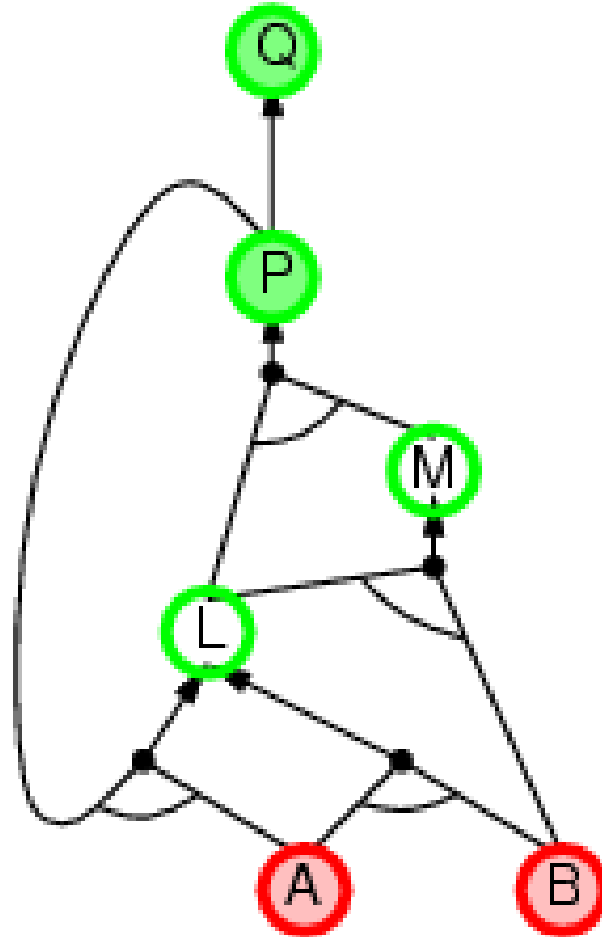
# Backward chaining example



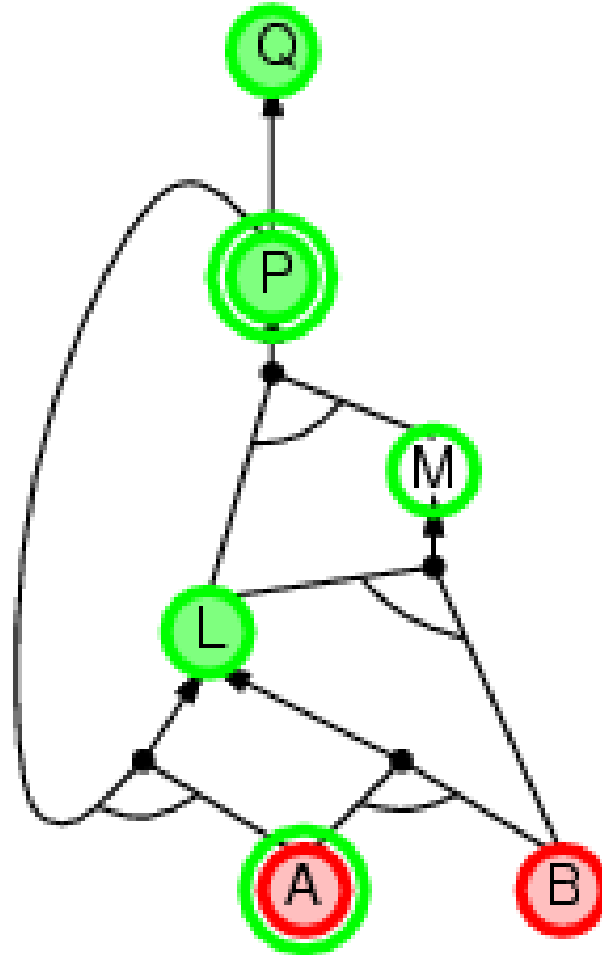
# Backward chaining example



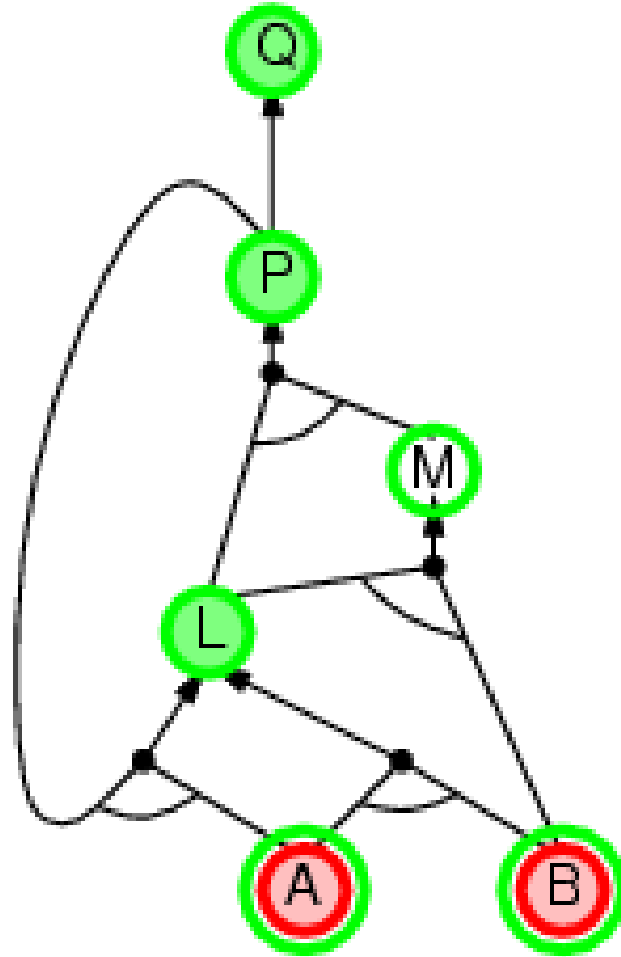
# Backward chaining example



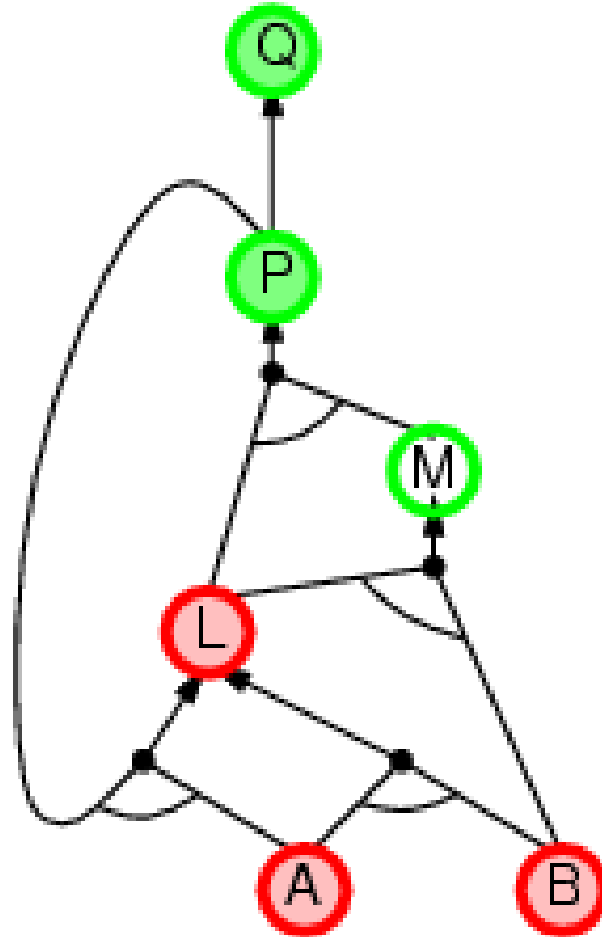
# Backward chaining example



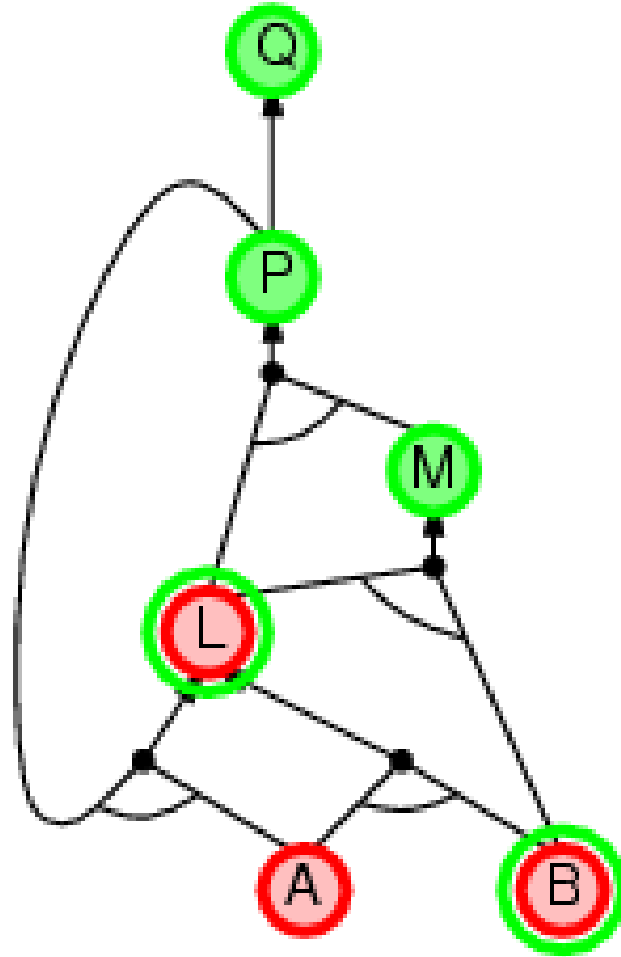
# Backward chaining example



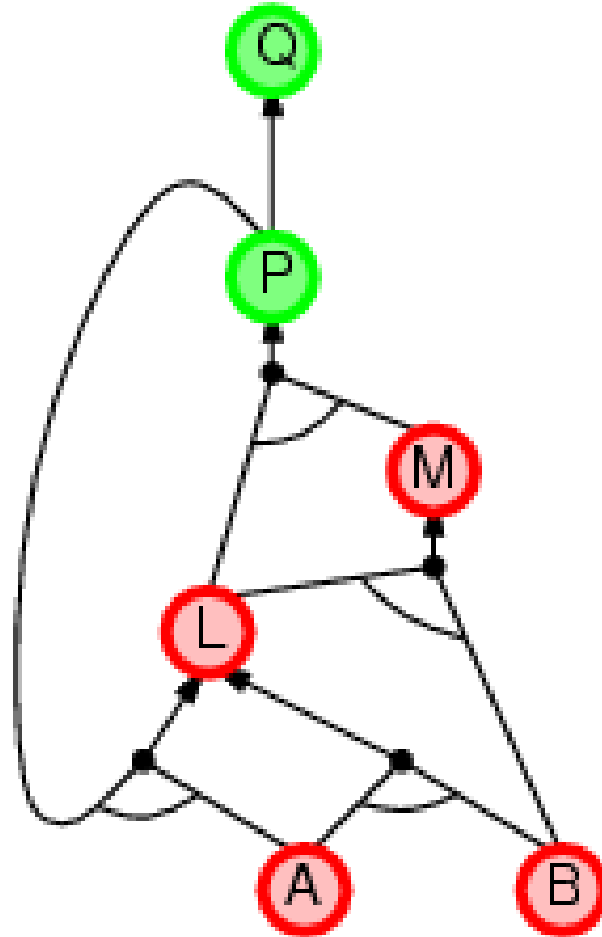
# Backward chaining example



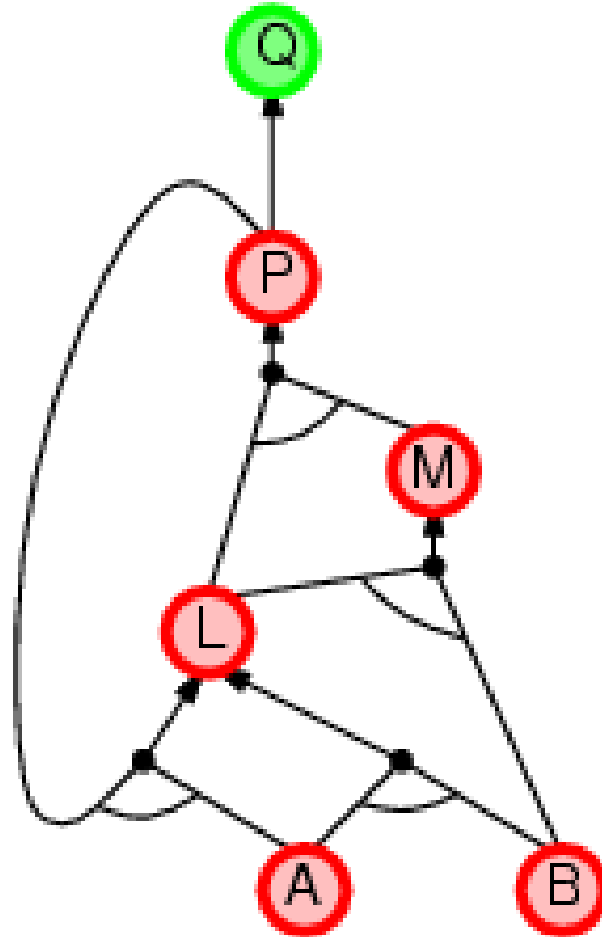
# Backward chaining example



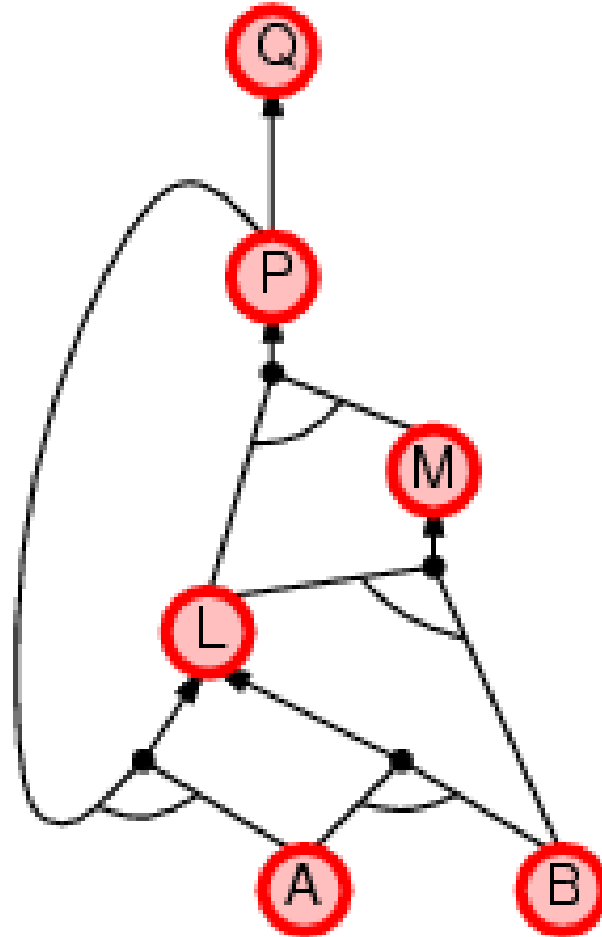
# Backward chaining example



# Backward chaining example



# Backward chaining example





# Forward vs. backward chaining

- FC is **data-driven**, automatic, unconscious processing,
  - e.g., object recognition, routine decisions
- May do lots of work that is irrelevant to the goal
- BC is **goal-driven**, appropriate for problem-solving,
  - e.g., Where are my keys? How do I get into a PhD program?
- Complexity of BC can be **much less** than linear in size of KB



# Efficient propositional inference

Two families of efficient algorithms for propositional inference:

Complete backtracking search algorithms

- DPLL algorithm (Davis, Putnam, Logemann, Loveland)
- Incomplete local search algorithms
  - WalkSAT algorithm



# The DPLL algorithm

Determine if an input propositional logic sentence (in CNF) is satisfiable.

Improvements over truth table enumeration:

1. Early termination

A clause is true if any literal is true.

A sentence is false if any clause is false.

2. Pure symbol heuristic

Pure symbol: always appears with the same "sign" in all clauses.

e.g., In the three clauses  $(A \vee \neg B)$ ,  $(\neg B \vee \neg C)$ ,  $(C \vee A)$ , A and B are pure, C is impure.

Make a pure symbol literal true.

3. Unit clause heuristic

Unit clause: only one literal in the clause

The only literal in a unit clause must be true.

# The DPLL algorithm

**function** DPLL-SATISFIABLE?(*s*) **returns** *true* or *false*

**inputs:** *s*, a sentence in propositional logic

*clauses* ← the set of clauses in the CNF representation of *s*

*symbols* ← a list of the proposition symbols in *s*

**return** DPLL(*clauses*, *symbols*, [])

---

**function** DPLL(*clauses*, *symbols*, *model*) **returns** *true* or *false*

**if** every clause in *clauses* is true in *model* **then return** *true*

**if** some clause in *clauses* is false in *model* **then return** *false*

*P*, *value* ← FIND-PURE-SYMBOL(*symbols*, *clauses*, *model*)

**if** *P* is non-null **then return** DPLL(*clauses*, *symbols*-*P*, [*P* = *value* | *model*])

*P*, *value* ← FIND-UNIT-CLAUSE(*clauses*, *model*)

**if** *P* is non-null **then return** DPLL(*clauses*, *symbols*-*P*, [*P* = *value* | *model*])

*P* ← FIRST(*symbols*); *rest* ← REST(*symbols*)

**return** DPLL(*clauses*, *rest*, [*P* = *true* | *model*]) **or**

DPLL(*clauses*, *rest*, [*P* = *false* | *model*])



# The WalkSAT algorithm

- Incomplete, local search algorithm
- Evaluation function: The min-conflict heuristic of minimizing the number of unsatisfied clauses
- Balance between greediness and randomness



# The WalkSAT algorithm

```
function WALKSAT(clauses, p, max-flips) returns a satisfying model or failure  
inputs: clauses, a set of clauses in propositional logic  
         p, the probability of choosing to do a “random walk” move  
         max-flips, number of flips allowed before giving up  
  
model ← a random assignment of true/false to the symbols in clauses  
for i = 1 to max-flips do  
    if model satisfies clauses then return model  
    clause ← a randomly selected clause from clauses that is false in model  
    with probability p flip the value in model of a randomly selected symbol  
        from clause  
    else flip whichever symbol in clause maximizes the number of satisfied clauses  
return failure
```



# Hard satisfiability problems

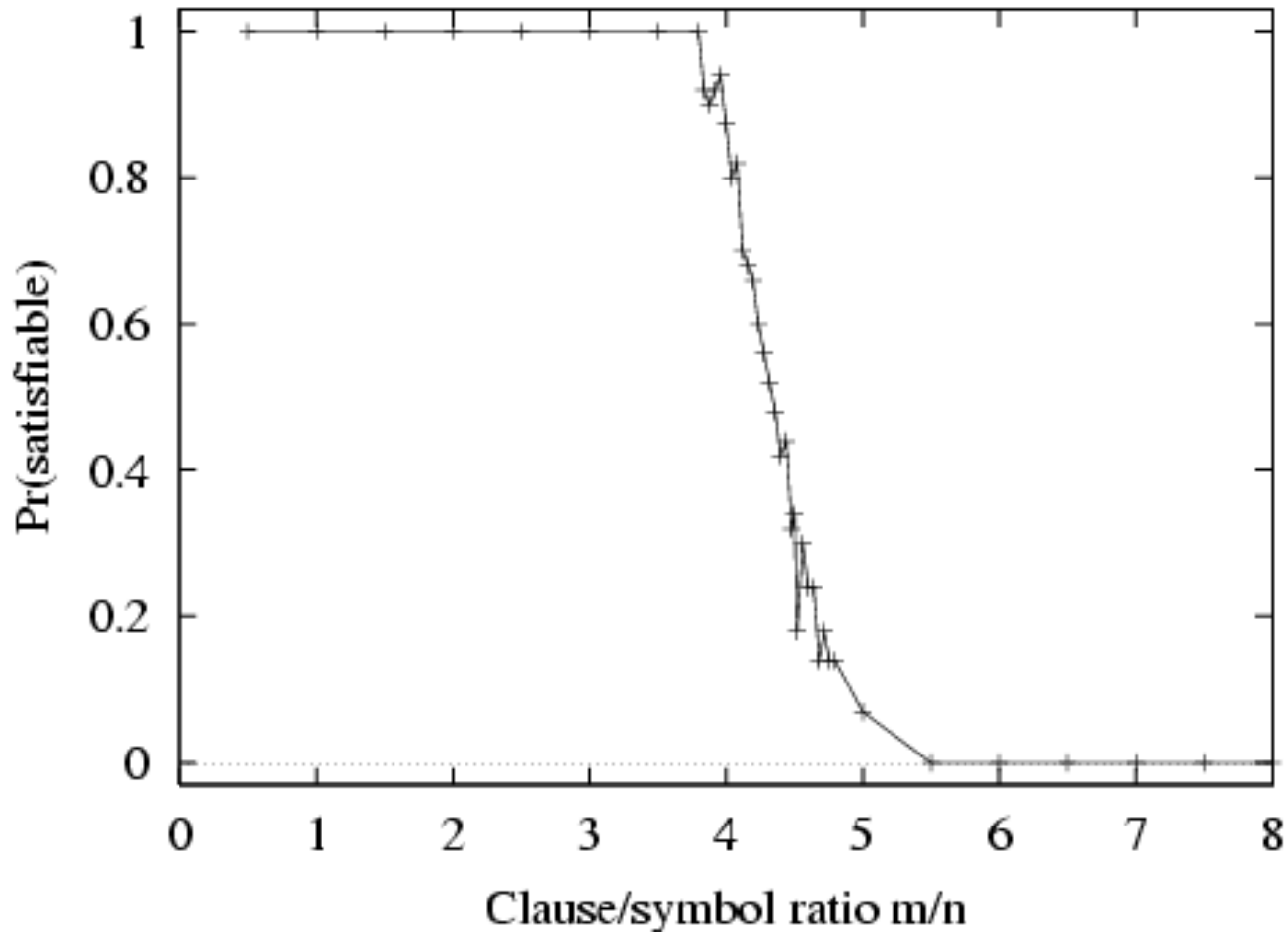
- Consider random 3-CNF sentences. e.g.,  
 $(\neg D \vee \neg B \vee C) \wedge (B \vee \neg A \vee \neg C) \wedge (\neg C \vee \neg B \vee E) \wedge (E \vee \neg D \vee B) \wedge (B \vee E \vee \neg C)$

$m$  = number of clauses

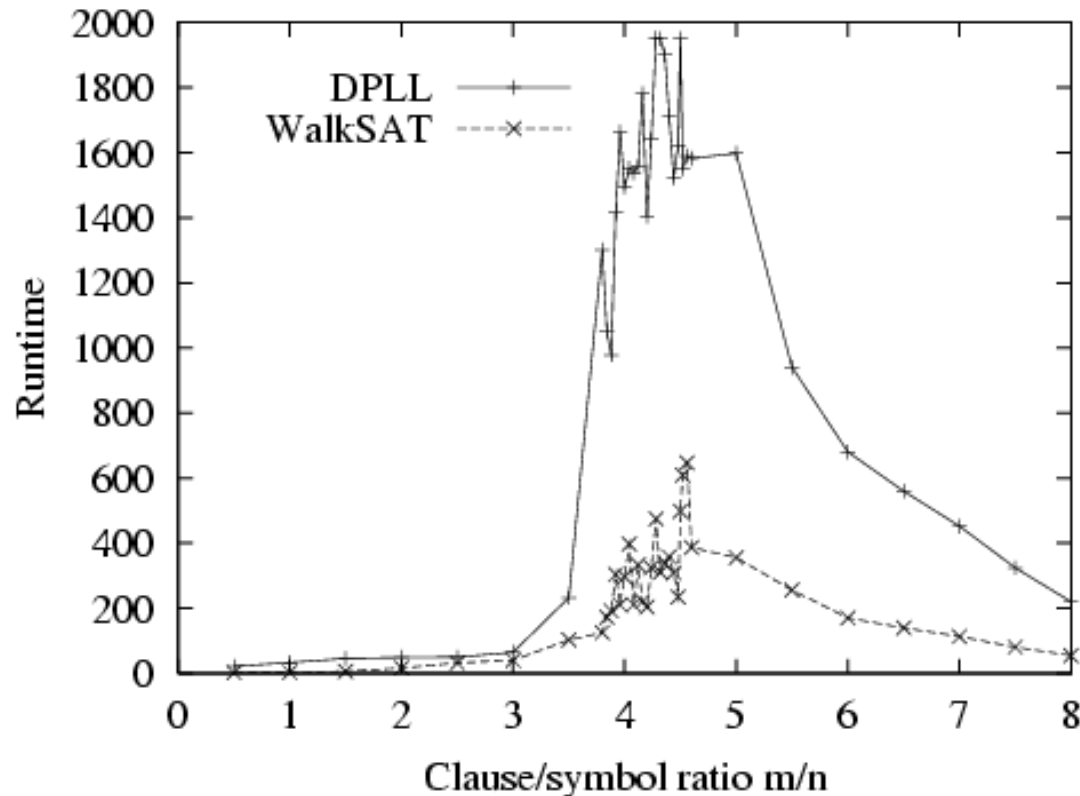
$n$  = number of symbols

- Hard problems seem to cluster near  $m/n = 4.3$   
(critical point)

# Hard satisfiability problems



# Hard satisfiability problems



- Median runtime for 100 **satisfiable** random 3-CNF sentences,  $n = 50$

# Inference-based agents in the wumpus world

A wumpus-world agent using propositional logic:

$$\begin{aligned} &\neg P_{1,1} \\ &\neg W_{1,1} \\ &B_{x,y} \Leftrightarrow (P_{x,y+1} \vee P_{x,y-1} \vee P_{x+1,y} \vee P_{x-1,y}) \\ &S_{x,y} \Leftrightarrow (W_{x,y+1} \vee W_{x,y-1} \vee W_{x+1,y} \vee W_{x-1,y}) \\ &W_{1,1} \vee W_{1,2} \vee \dots \vee W_{4,4} \\ &\neg W_{1,1} \vee \neg W_{1,2} \\ &\neg W_{1,1} \vee \neg W_{1,3} \\ &\dots \square \end{aligned}$$

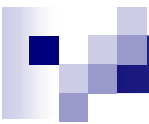
$\Rightarrow$  64 distinct proposition symbols, 155 sentences

```

function PL-WUMPUS-AGENT(percept) returns an action
  inputs: percept, a list, [stench,breeze,glitter]
  static: KB, initially containing the “physics” of the wumpus world
           x, y, orientation, the agent’s position (init. [1,1]) and orient. (init. right)
           visited, an array indicating which squares have been visited, initially false
           action, the agent’s most recent action, initially null
           plan, an action sequence, initially empty

  update x,y,orientation, visited based on action
  if stench then TELL(KB,  $S_{x,y}$ ) else TELL(KB,  $\neg S_{x,y}$ )
  if breeze then TELL(KB,  $B_{x,y}$ ) else TELL(KB,  $\neg B_{x,y}$ )
  if glitter then action  $\leftarrow$  grab
  else if plan is nonempty then action  $\leftarrow$  POP(plan)
  else if for some fringe square [i,j], ASK(KB, ( $\neg P_{i,j} \wedge \neg W_{i,j}$ )) is true or
           for some fringe square [i,j], ASK(KB, ( $P_{i,j} \vee W_{i,j}$ )) is false then do
           plan  $\leftarrow$  A*-GRAPH-SEARCH(ROUTE-PB([x,y], orientation, [i,j], visited))
           action  $\leftarrow$  POP(plan)
  else action  $\leftarrow$  a randomly chosen move
  return action

```



# Expressiveness limitation of propositional logic

- KB contains "physics" sentences for every single square
- For every time  $t$  and every location  $[x,y]$ ,  
 $L_{x,y}^t \wedge \textit{FacingRight}^t \wedge \textit{Forward}^t \Rightarrow L_{x+1,y}^t$
- Rapid proliferation of clauses



# Summary

- Logical agents apply **inference** to a **knowledge base** to derive new information and make decisions
- Basic concepts of logic:
  - **syntax**: formal structure of **sentences**
  - **semantics**: **truth** of sentences wrt **models**
  - **entailment**: necessary truth of one sentence given another
  - **inference**: deriving sentences from other sentences
  - **soundness**: derivations produce only entailed sentences
  - **completeness**: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Resolution is complete for propositional logic  
Forward, backward chaining are linear-time, complete for Horn clauses
- Propositional logic lacks expressive power