Introduction to Software Testing
Chapter 3.6
Disjunctive Normal Form Criteria

Paul Ammann & Jeff Offutt

http://www.cs.gmu.edu/~offutt/softwaretest/

Disjunctive Normal Form

- **Common Representation for Boolean Functions**
  - **Slightly Different Notation for Operators**
  - **Slightly Different Terminology**
- **Basics:**
  - **A *literal* is a clause or the negation (overstrike) of a clause**
    - **Examples:** $a, \overline{a}$
  - **A *term* is a set of literals connected by logical "and"**
    - **"and" is denoted by adjacency instead of** $\wedge$
    - **Examples:** $ab, a\overline{b}, \overline{a}\overline{b}$ **for** $a \wedge b, a \wedge \neg b, \neg a \wedge \neg b$
  - **A *(disjunctive normal form) predicate* is a set of terms connected by "or"**
    - **"or" is denoted by** $+$ **instead of** $\vee$
    - **Examples:** $abc + \overline{a}b + a\overline{c}$
    - **Terms are also called "implicants"**
      - **If a term is true, that implies the predicate is true**

Implicant Coverage

- **Obvious coverage idea: Make each implicant evaluate to "true".**
  - **Problem: Only tests "true" cases for the predicate.**
  - **Solution: Include DNF representations for negation.**

<u>Implicant Coverage (IC)</u> : Given DNF representations of a predicate *f* and its negation $\overline{f}$, for each implicant in *f* and $\overline{f}$, TR contains the requirement that the implicant evaluate to true.

- **Example:** $f = ab + b\overline{c}$ $\quad\quad \overline{f} = \overline{b} + \overline{a}c$
  - **Implicants:** $\{ ab, b\overline{c}, \overline{b}, \overline{a}c \}$
  - **Possible test set: {TTF, FFT}**
- **Observation: IC is relatively weak**

Improving on Implicant Coverage

- **Additional Definitions:**
  - **A *proper subterm* is a term with one or more clauses removed**
    - **Example: *abc* has 6 proper subterms: *a, b, c, ab, ac, bc***
  - **A *prime implicant* is an implicant such that no proper subterm is also an implicant.**
    - **Example:** $f = ab + a\bar{b}c$
    - **Implicant *ab* is a prime implicant**
    - **Implicant** $a\bar{b}c$ **is not a prime implicant (due to proper subterm *ac*)**
  - **A *redundant implicant* is an implicant that can be removed without changing the value of the predicate**
    - **Example:** $f = ab + ac + b\bar{c}$
    - ***ab* is redundant**
    - **Predicate can be written:** $ac + b\bar{c}$

Unique True Points

- A ***minimal DNF representation*** is one with only prime, nonredundant implicants.
- A ***unique true point*** with respect to a given implicant is an assignment of truth values so that
  - the given implicant is true, and
  - all other implicants are false
- Hence a unique true point test focuses on just one implicant
- A minimal representation guarantees the existence of at least one unique true point for each implicant

Unique True Point Coverage (UTPC) : Given minimal DNF representations of a predicate *f* and its negation $\bar{f}$, TR contains a unique true point for each implicant in *f* and $\bar{f}$.

Unique True Point Example

- **Consider again:** $f = ab + b\overline{c}$ $\quad \overline{f} = \overline{b} + \overline{a}c$
  - **Implicants:** $\{ ab, b\overline{c}, \overline{b}, \overline{a}c \}$
  - **Each of these implicants is prime**
  - **None of these implicants is redundant**
- **Unique true points:**
  - ab**:** **{TTT}**
  - $b\overline{c}$**:** **{FTF}**
  - $\overline{b}$**:** **{FFF, TFF, TFT}**
  - $\overline{a}c$**:** **{FTT}**
- **Note that there are three possible (minimal) tests satisfying UTPC**
- **UTPC is fairly powerful**
  - **Exponential in general, but reasonable cost for many common functions**
  - **No subsumption relation wrt any of the ACC or ICC Criteria**

Near False Points

- A *near false point* with respect to a clause *c* in implicant *i* is an assignment of truth values such that *f* is false, but if *c* is negated (and all other clauses left as is), *i* (and hence *f*) evaluates to true.
- Relation to *determination*: at a near false point, *c* determines *f*
  - Hence we should expect relationship to ACC criteria

Unique True Point and Near False Point Pair Coverage (CUTPNFP) : Given a minimal DNF representation of a predicate *f*, for each clause *c* in each implicant *i*, TR contains a unique true point for *i* and a near false point for *c* such that the points differ only in the truth value of *c*.

- Note that definition only mentions *f*, and not $\overline{f}$.
- Clearly, CUTPNFP subsumes RACC

CUTPNFP Example

- **Consider *f = ab + cd***
  - **For implicant *ab*, we have 3 unique true points: {TTFF, TTFT, TTTF}**
    - **For clause *a*, we can pair unique true point TTFF with near false point FTFF**
    - **For clause b, we can pair unique true point TTFF with near false point TFFF**
  - **For implicant *cd*, we have 3 unique true points: {FFTT, FTTT, TFTT}**
    - **For clause *c*, we can pair unique true point FFTT with near false point FFFT**
    - **For clause *d*, we can pair unique true point FFTT with near false point FFTF**
- **CUTPNFP set: {TTFF, FFTT, TFFF, FTFF, FFTF, FFFT}**
  - **First two tests are unique true points; others are near false points**
- **Rough number of tests required: # implicants * # literals**

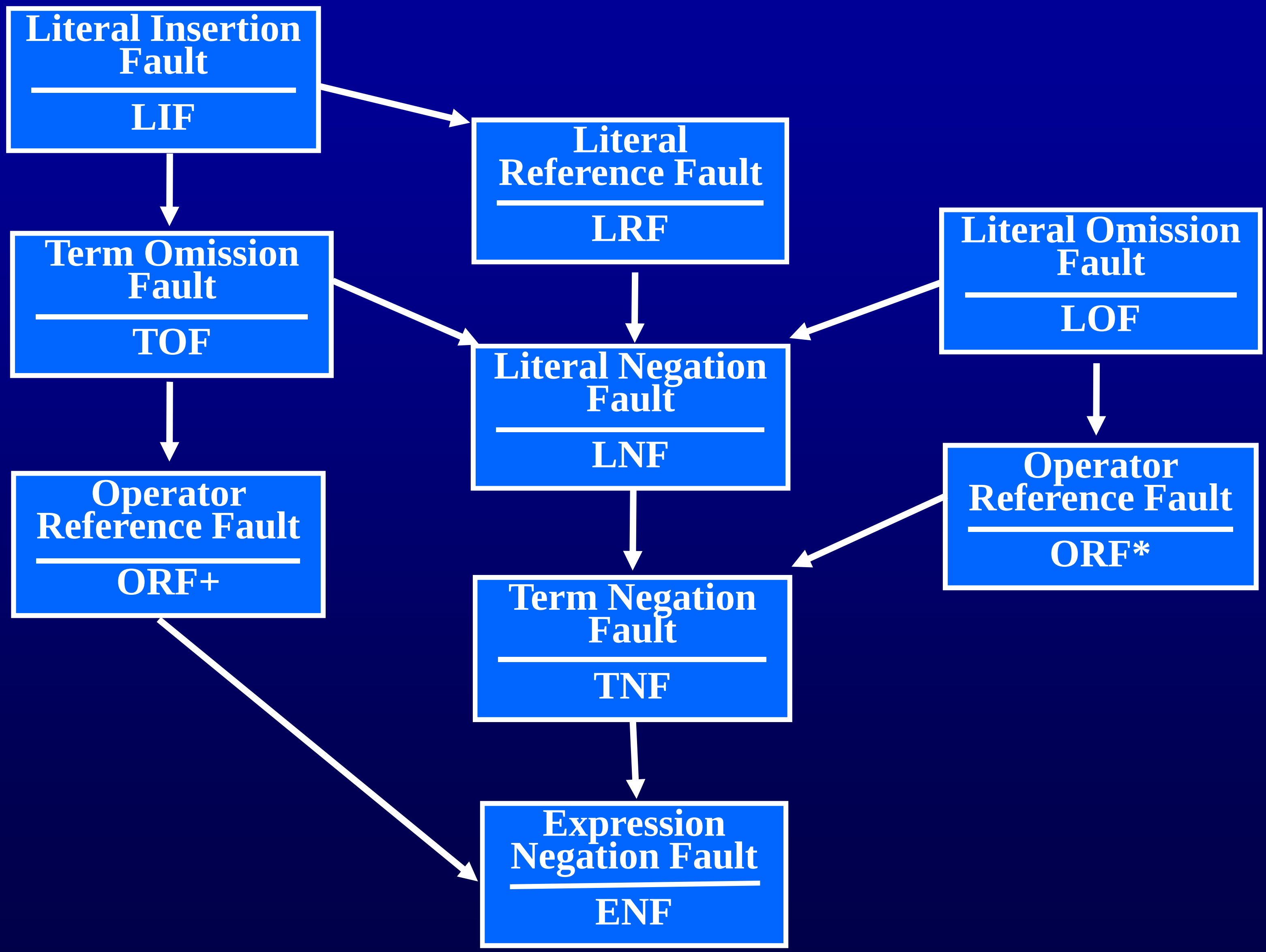
DNF Fault Classes

- **ENF: Expression Negation Fault** $f = ab+c$ $f' = \overline{ab+c}$
- **TNF: Term Negation Fault** $f = ab+c$ $f' = \overline{ab}+c$
- **TOF: Term Omission Fault** $f = ab+c$ $f' = ab$
- **LNF: Literal Negation Fault** $f = ab+c$ $f' = a\overline{b}+c$
- **LRF: Literal Reference Fault** $f = ab + bcd$ $f' = ad + bcd$
- **LOF: Literal Omission Fault** $f = ab + c$ $f' = a + c$
- **LIF: Literal Insertion Fault** $f = ab + c$ $f' = ab + bc$
- **ORF+: Operator Reference Fault** $f = ab + c$ $f' = abc$
- **ORF*: Operator Reference Fault** $f = ab + c$ $f' = a + b + c$

Key idea is that fault classes are related with respect to testing:

Test sets guaranteed to detect certain faults are also guaranteed to detect additional faults.

Fault Detection Relationships



Understanding The Detection Relationships

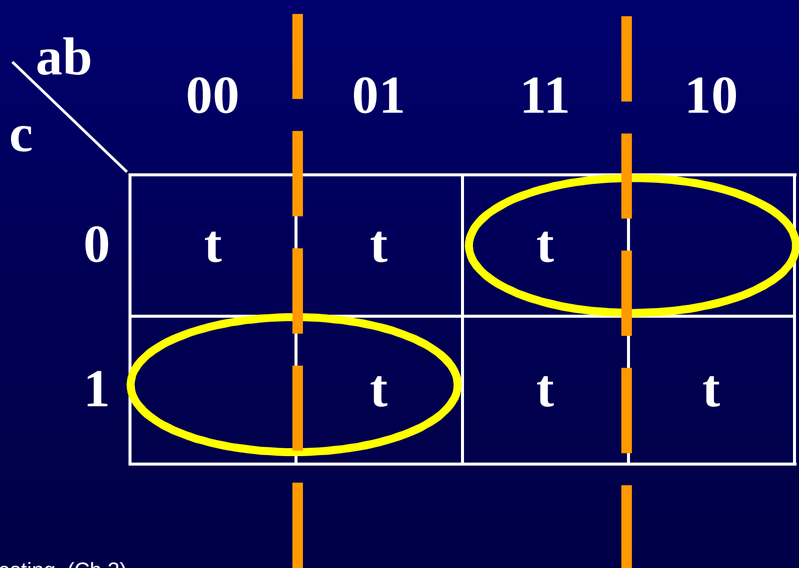
- Consider the TOF (Term Omission Fault) class
  - UTPC requires a unique true point for every implicant (term)
  - Hence UTPC detects all TOF faults
  - From the diagram, UTPC also detects:
    - All LNF faults (Unique true point for implicant now false)
    - All TNF faults (All true points for implicant are now false points)
    - All ORF+ faults (Unique true points for joined terms now false)
    - All ENF faults (Any single test detects this…)
- Although CUTPNFP does not subsume UTPC, CUTPNFP detects all fault classes that UTPC detects (Converse is false)
- Consider what this says about the notions of subsumption vs. fault detection
- Literature has many more powerful (and more expensive) DNF criteria
  - In particular, possible to detect entire fault hierarchy (MUMCUT)

Karnaugh Maps for Testing Logic Expressions

- **Fair Warning**
  - **We *use*, rather than *present*, Karnaugh Maps**
  - **Newcomer to Karnaugh Maps probably needs a tutorial**
    - **Suggestion: Google "Karnaugh Map Tutorial"**
- **Our goal: Apply Karnaugh Maps to concepts used to test logic expressions**
  - **Identify when a clause determines a predicate**
  - **Identify the negation of a predicate**
  - **Identify prime implicants and redundant implicants**
  - **Identify unique true points**
  - **Identify unique true point / near false point pairs**
- **No new material here on *testing***
  - **Just fast shortcuts for concepts already presented**

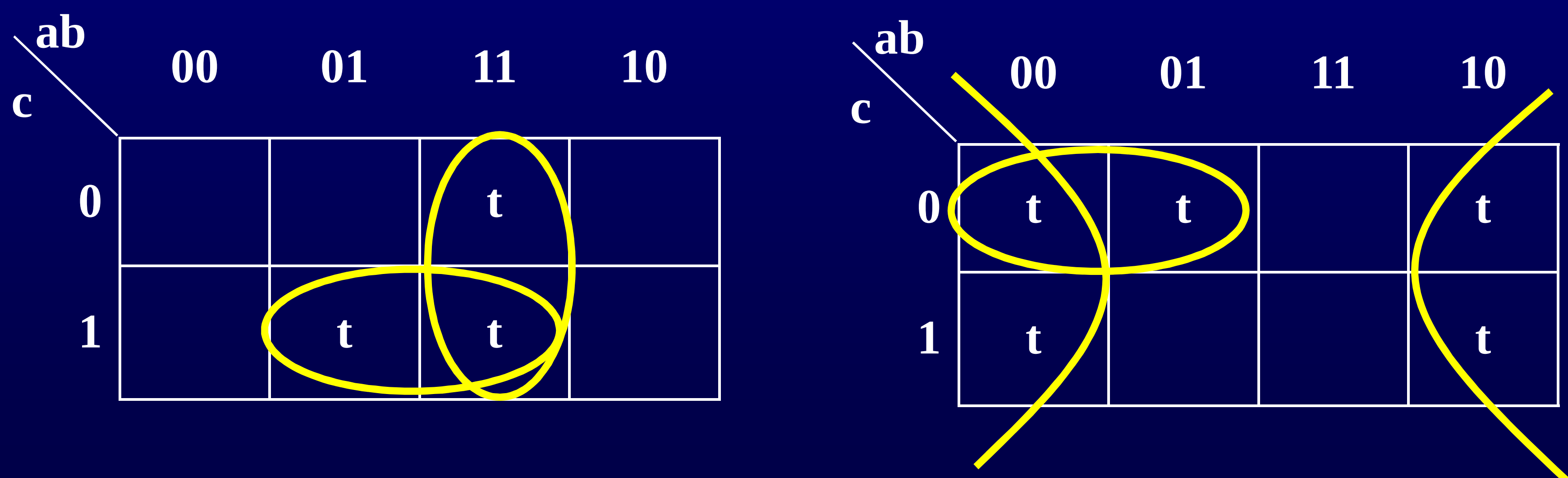
K-Map: A clause determines a predicate

- **Consider the predicate: $f = b + \bar{a}\bar{c} + ac$**
- **Suppose we want to identify when b determines f**
- **The dashed line highlights where b changes value**
  - **If two cells joined by the dashed line have different values for f, then b determines f for those two cells.**
  - **b determines f: $\bar{a}c + a\bar{c}$ (but NOT at ac or $\bar{a}\bar{c}$)**
- **Repeat for clauses a and c**



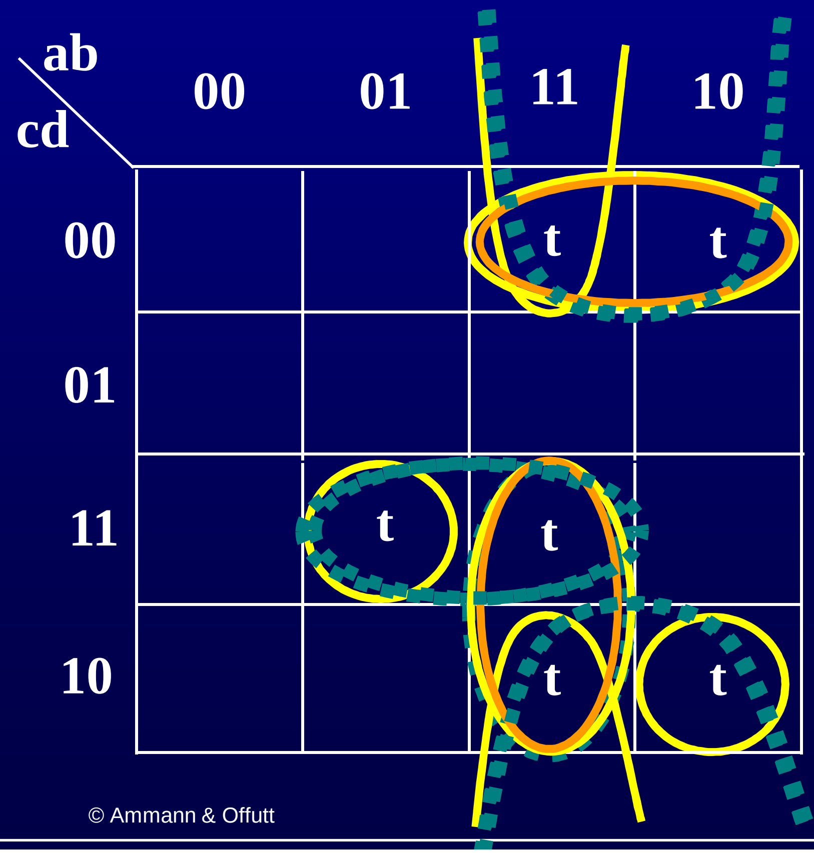
K-Map: Negation of a predicate

- **Consider the predicate: $f = ab + bc$**
- **Draw the Karnaugh Map for the negation**
  - **Identify groups**
  - **Write down negation: $\overline{f} = \overline{b} + \overline{a}\,\overline{c}$**



K-Map: Prime and redundant implicants

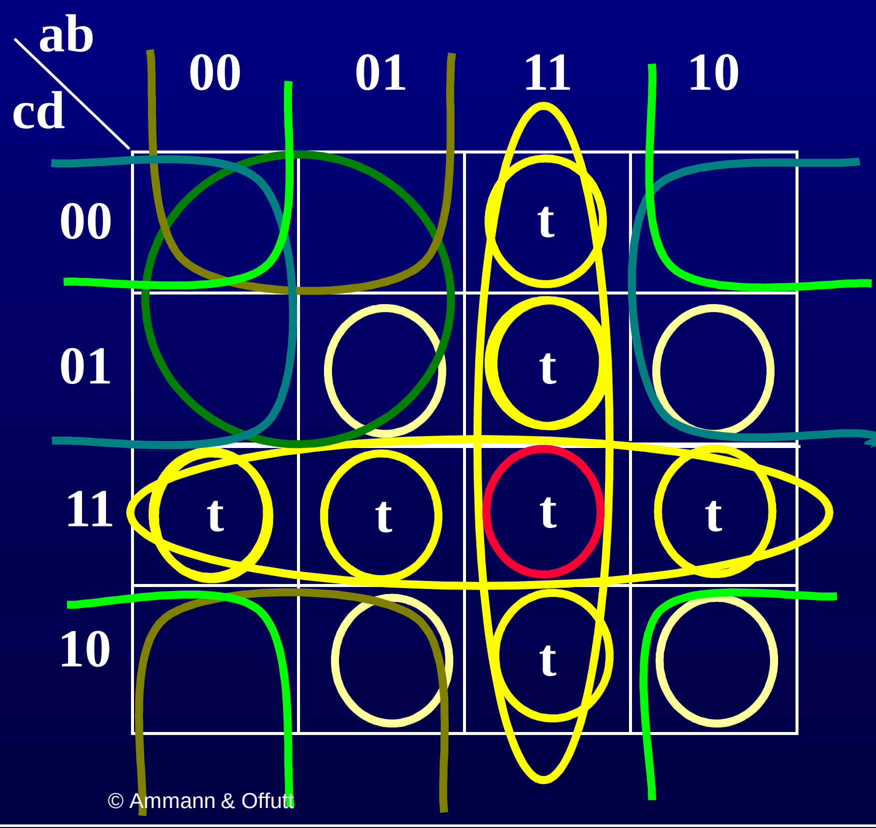
- **Consider the predicate:** $f = abc + ab\bar{d} + \bar{a}bcd + a\bar{b}c\bar{d} + a\bar{c}\bar{d}$
- **Draw the Karnaugh Map**
- **Implicants that are not prime:** $ab\bar{d}$, $\bar{a}bcd$, $a\bar{b}c\bar{d}$, $a\bar{c}\bar{d}$
- **Redundant implicant:** $ab\bar{d}$
- **Prime implicants**
  - **Three:** $a\bar{d}$, bcd, abc
  - **The last is redundant**
  - **Minimal DNF representation**
    - $f = a\bar{d} + bcd$



K-Map: Unique True Points

- **Consider the predicate: $f = ab + cd$**
- **Three unique true points for *ab***
  - **TTFF, TTFT, TTTF**
  - **TTTT is a true point, but not a unique true point**
- **Three unique true points for *cd***
  - **FFTT, FTTT, TFTT**
- **Unique true points for $\overline{f}$**

  $\overline{f} = \overline{a}\,\overline{c} + \overline{b}\,\overline{c} + \overline{a}\,\overline{d} + \overline{b}\,\overline{d}$
  - **FTFT,TFFT, FTTF, TFTF**
- **Possible UTPC test set**
  - ***f*: {TTFT, FFTT}**
  - **$\overline{f}$: {FTFT, TFFT, FTTF, TFTF}**



K-Map: Unique True Point/ Near False Point Pairs

- **Consider the predicate: *f* = *ab* + *cd***
- **For implicant *ab***
  - **For *a*, choose UTP, NFP pair**
    - **TTFF, FTFF**
  - **For *b*, choose UTP, NFP pair**
    - **TTFT, TFFT**
- **For implicant cd**
  - **For *c*, choose UTP, NFP pair**
    - **FFTT, FFFT**
  - **For *d*, choose UTP, NFP pair**
    - **FFTT, FFTF**
- **Possible CUTPNFP test set**
  - **{TTFF, TTFT, FFTT //UTPs**
    FTFF, TFFT, FFFT, FFTF} //NFPs

cd \ ab	00	01	11	10
00			t	
01			t	
11	t	t	t	t
10			t	