Performance of CSMA/CD

- As before, $U = 1/(1+a)$, where $a = RD/LV$.
- For an IEEE 802.3, $R = 10^7$, $D = 2500\,m$, and $V = 2 \times 10^8 m/s$.
- Given N nodes with a probability of sending of p, there are three possible outcomes:
  - No node sends: $P(0) = \frac{N!}{0!N!}p^0(1-p)^N$
  - One node sends: $P(1) = \frac{N!}{1!(N-1)!}p^1(1-p)^{N-1}$
  - A collision occurs: $P(>1) = 1 - P(0) - P(1)$
- Let A be the probability that a station successfully sends.

- To make it simple, the maximum value for P(1) is when $p = 1/N$ and $A = (1 - 1/N)^{N-1}$
- As N gets large, $A \to e^{-1}$.
- The expected number of slots needed to send a frame is found from the mean of the geometric distribution to be: $1/A$ and the expected number of failures is $1/A - 1$.
- As N get large, $A \to e^{-1}$ and the expected number of failures goes to 1.72.

- Then the utilization is:
  $$U = \frac{FrameTime}{FrameTime + Overhead} = \frac{L/R}{(L/R + D/V) + 2\tau\ (1/A - 1)}.$$
- $$U = \frac{L/R}{(L/R + D/V) + 2(D/V)\ (1/A - 1)}.$$
- $$U = \frac{1}{(1 + a) + 2a/(1/A - 1}.$$
- $$U = \frac{1}{1 + a + 2a(1-A)/A}.$$
- When N gets large, $U \rightarrow \frac{1}{1+4.44a}$

Utilization and Load

If a = 2.44 ($L = 512b$), as N grows, the utilization for CSMA/CD is

N	A	U
1	1	0.29
2	0.5	0.12
3	0.45	0.11
5	0.41	0.095
10	0.39	0.090
20	0.38	0.087
50	0.37	0.085

As N gets large, $A \rightarrow 0.367$, $U \rightarrow 0.084$ for the worst case value for a.

Reality Check

This is a theoretical result and indicates that as N grows large, the utilization approaches a limit asymptotically. While this may be true, the problem for Ethernets is that throughput drops off dramatically as the number of collisions increases. As A gets large, you can reach a saturation point where messages begin to backlog. Then the probability of a node having a message to send starts to increase, which increases A. As the number of contention periods needed to send increases, so does the backlog and so on.