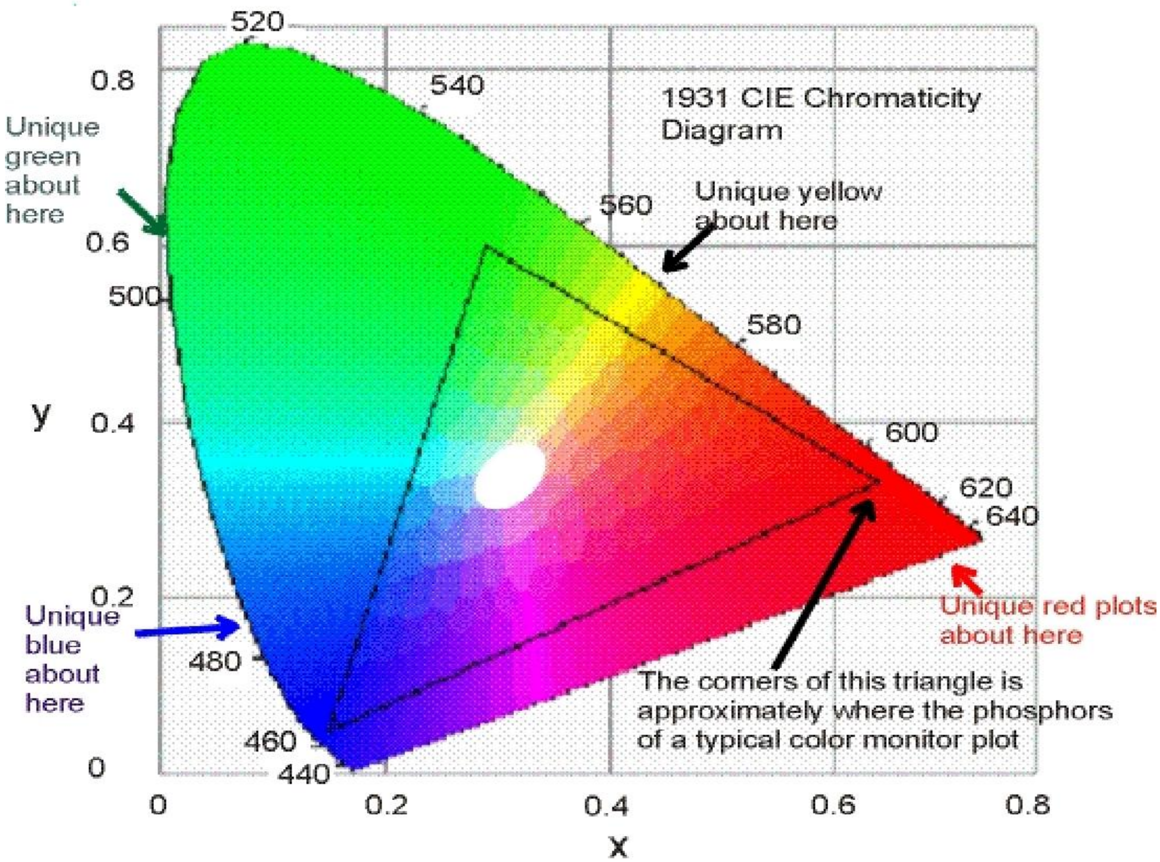
The CIE Chromaticity Diagram

Major points of these notes:

1. Background of the CIE Diagram
2. The CIE Diagram Axes
3. Using the CIE Diagram

Background of the CIE Diagram

The CIE diagram, or more precisely the CIE Chromaticity Diagram, is shown below. It was created in 1931 by the Commission Internationale de l'Eclairage.



Visually it is a curved triangle that contains the full range of visible colors. Some things that you should notice, to work with the diagram, are:

- White surrounds the center point, named *C*, which has coordinates at about (0.333, 0.333).
- The *x* and *y* axes both range from 0 to close to 0.8.
- Wavelengths from about 380 to 700 follow around the curved edge.
- This version claims to show the range of a typical monitor, with the corners of the monitor triangle showing where full intensity red, green, and blue are likely to occur. As we discuss later, the monitor shown is a particularly bad one. The common claim for a good monitor is that red is at about (.735. .265), green at about (.274, .717), and blue at about (.167, .009).

The CIE Diagram Axes

Before I get into the uses of the diagram I will first get into the way that the axes are defined.

The CIE Diagram is an additive color system based on three color primaries named X, Y, and Z. Obviously the diagram only uses two dimensions, which are x and y selected from x, y, and z, where:

$$x = \frac{X}{X+Y+Z}, \quad y = \frac{Y}{X+Y+Z}, \text{ and } z = \frac{Z}{X+Y+Z}.$$

Since $x + y + z = 1$, we don't need to include z because it can be computed from x and y using $z = 1 - x - y$.

We've taken three dimensions of color information, (X, Y, Z) and have now managed to plot it using only two dimensions, (x, y). Obviously we must have lost something. The part that we have lost is the intensity of the color. To show this, consider two color specifications (X, Y, Z) and (aX, aY, aZ) for any constant a. Both will be plotted at the same (x, y) location on the diagram because

$$\frac{aX}{aX+aY+aZ} = \frac{X}{X+Y+Z} \quad \text{and} \quad \frac{aY}{aX+aY+aZ} = \frac{Y}{X+Y+Z}$$

Using the CIE Diagram

Mixing colors: If we choose two points on the diagram and draw a straight line between them, we will pass through all colors that can be formed by mixing those colors. As a result, if we select three points on the diagram then the triangle defined by the three points will give all colors that can be formed by mixing the three colors.

Evaluating monitors: If we know where a specific monitor's red, green, and blue phosphors fall on the CIE diagram, the mixing property shows the full range of colors that can be generated by the monitor. Looking at the diagram above, which showed this triangle for a relatively poor monitor, this is very depressing. It tells us that some monitors can only display about half of the visible colors. Even if we use a better monitor with phosphors at the more common (.735, .265), (.274, .717), and (.167, .009), things aren't much better. To get further depressed, no monitor will ever be able to generate all possible colors, because of the curved sides in the diagram. To include all visible colors the phosphor would have to fall outside of the diagram, and so unmixed use of that phosphor would be invisible, which would be even worse. When purchasing a monitor, a better monitor will have phosphors that push further into the corners of the CIE triangle, which will let you generate more colors. The range of colors is called the monitor's color gamut. www.cs.rit.edu/~ncs/color/a_chroma.html has an applet that lets you play with a color gamut and color mixes.

Complementary colors: Two colors are complementary if they can be mixed to produce white. In terms of the diagram, this means that to find the complement of a given color draw a line through the middle of the white, and colors on the far side are complementary to your given color. The center of white, which is at about (.333, .333), is defined to be the color of sunlight.

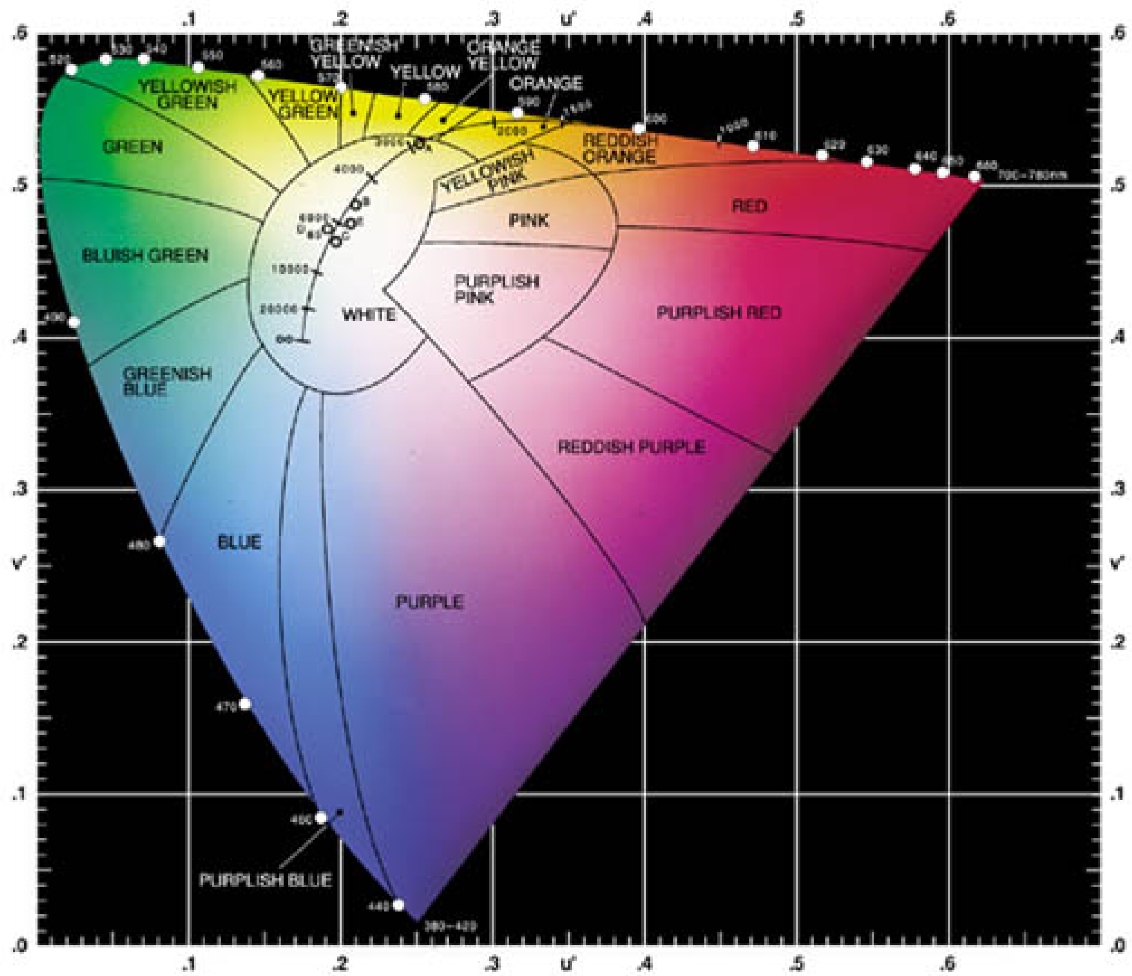
Pure colors: These are on the edges of the diagram. Pure complementary colors are on edges opposite white.

Purity and dominant wavelength (chromaticity): The chromaticity of a color is a combination of its purity and its dominant wavelength. Given a point on the diagram, draw a line through the point from the center, *C*, to the edge. The dominant wavelength or hue of the color is the wavelength of the pure color where the line meets the edge. The purity is expressed as a percentage of how far down the line that the point lies. Points with a high purity lie close to the edge, and those with a low purity lie close to *C*.

Conversion to RGB value: This conversion depends on where the RGB phosphor lies on the CIE diagram for your monitor. Fortunately typical phosphor values can be used, and the conversions below, which I've been using above, seem to be most commonly quoted:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} .735 & .265 & .000 \\ .274 & .717 & .009 \\ .167 & .009 & .824 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}.$$

The Commission defines color areas of the diagram by graphically defining regions.



The diagram above is intended to show two things. One is that there are different versions of the diagram available. This version is a rescaling of the traditional CIE diagram done in 1976 to give smoother color changes. Another is that areas can be defined, at different granularity, on any CIE diagram which name color regions. This diagram has large color areas and so less color regions defined. Note that even at this granularity there is a difference between say, purplish-red and reddish-purple.

Now say that we want to use the diagram to display a reddish-orange color using the CIE definitions. We can find a traditional CIE Diagram (like our original one) with the color regions marked, and decide that the center of the reddish-orange region is at (.47, .51). To get the RGB value we first convert this into (x, y, z) by remembering that $x + y + z = 1$, which gives us the triple (.47, .51, .02). Since these add to 1 we can also use them as our (X, Y, Z) triple, and put them into the matrix conversion equation that we gave above,

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} .735 & .265 & .000 \\ .274 & .717 & .009 \\ .167 & .009 & .824 \end{bmatrix} \begin{bmatrix} .47 \\ .51 \\ .02 \end{bmatrix}$$

This will give an RGB value of (.480, .637, .098) for reddish-orange. Given the equivalence under multiplication of the CIE diagram any multiple of these values (where all stay below 1.0) will be equivalent in terms of chromaticity.

I’ve tried using this color selection approach and haven’t been very satisfied with the results, but others claim better satisfaction.