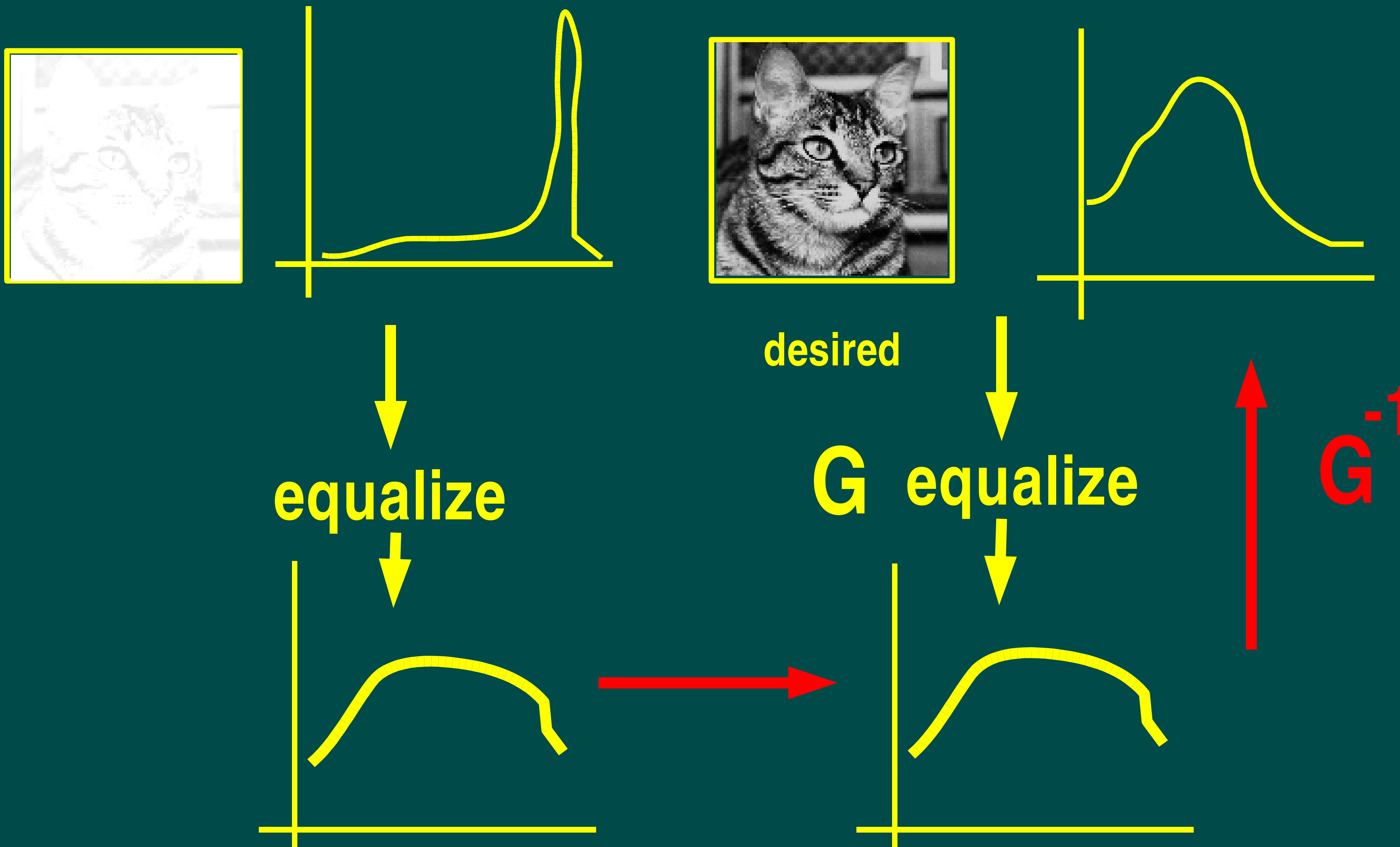
Histogram Specification



Histogram Specification (cont.)

- Equalize the levels of the original image.
- Specify the desired density function (histogram) and obtain the transformation function G(z).
- Apply the inverse transformation function $z = G^{-1}(s)$ to the levels obtained in first step.

Histogram Specification (cont.) -from text-

- Saw what we want the histogram to look like and come up with a transform function that will give it to us.
- Continuous random variables r & z. Pr(r) and Pz(z) denote their probability density functions. (Continuous equivalent of a histogram).
- Pr(r) input image
- Pz(z) desired output image (processed) This is what we'd like the processed image to have.

Histogram Specification (cont.)
-from text-

- Continuous version of histogram equalization.
- Cumulative Distribution Function

$$s = T(r) = \int_0^r Pr(w)\,dw$$

Histogram Specification (cont.) -from text-

- Continuous version of histogram equalization.
- Cumulative distribution function.

$$s = T(r) = \int_0^r P_r(w)\,dw$$

$$G(z) = \int_0^z P_z(t)\,dt = s$$

$$\rightarrow G(z) = T(r)$$

$$z = G^{-1}(s) = G^{-1}(T(r))$$

Histogram Specification (cont.) -from text-

- If you have Pr(r) estimated from input image, you can obtain T(r) from

$$\int_0^r P_r(w)\,dw$$

Transformation G(z) can be obtained by using

$$\int_0^z P_z(t)\,dt$$

since you specified Pz(z)!

Histogram Specification (cont.) -from text-

- Assume G^{-1} exists and that is satisfys:
- Single-Valued, monotonically increasing
- $0 <= G^{-1}(z) <= 1$ for $0 <= z <= 1$

Histogram Specification (cont.) -from text-

- You can obtain an image with the specified probability density function from an input function using the following:
  - obtain T(r) using equilization
  - obtain G(z) by equalizing specified histogram
  - obtain $G^{-1}(z)$
  - obtain the output image by applying G^{-1} to all pixels in input image.

Histogram Specification (cont.) -from text-

- So far so good (continuously speaking)
- In practice it is difficult to obtain analytical expressions for T(r) and G^{-1}
- With discrete values it becomes possible to make a close approximation to the histogram.

discrete Formulas

$$s_k = T(r_k) = \sum_{j=0}^{k} P_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \quad k = 0,1,2,\ldots,L-1$$

L is the number of discrete gray levels

n = total # pixels

n_j = # of pixels with gray level j

$$v_k = G(z_k) = \sum_{i=0}^{k} p_z(z_i) = s_k \quad k = 0,1,2,\ldots,L-1$$

$$z_k = G^{-1}[T(r_k)] \quad k = 0,1,2,\ldots,L-1$$

$$z_k = G^{-1}(s_k) \quad k = 0,1,2,\ldots,L-1$$

Implementation

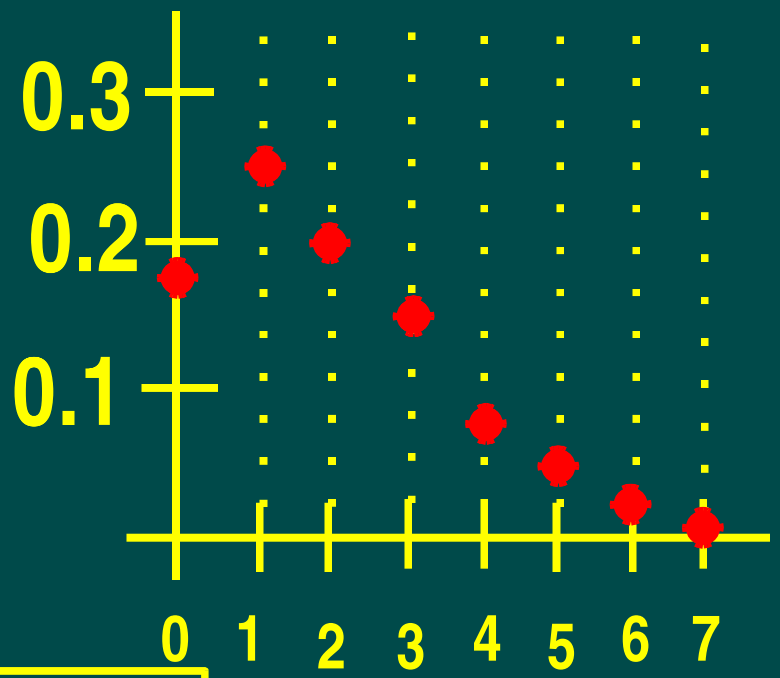
- Each set of gray levels is a 1D array
- All mappings from r to s and s to z are simple table lookups
- Each element (e.g. s_k) contains 2 important pieces of information:
  - subscript k denotes the location of the element in the array
  - s denotes the value at that location
- We need to only look at integer values

Refer to Figure 3.19 GW

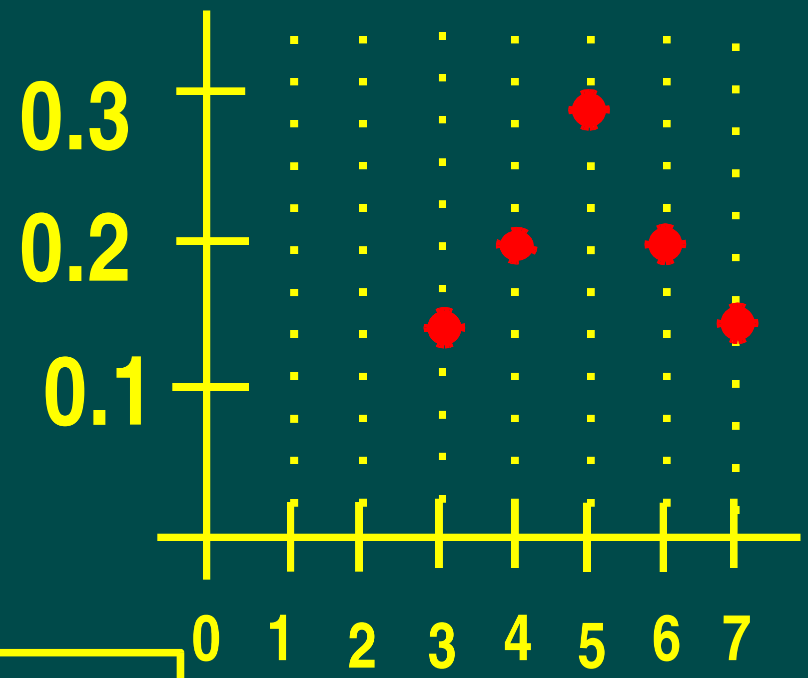
- (a) a hypothetical transform function given an image that was equalized.
- (b) Equalize specified histogram. G^{-1} is just running the transform backwards.
- But wait a minute! Where did we get the z's???
- We have to use an iterative scheme.

Iterative Scheme

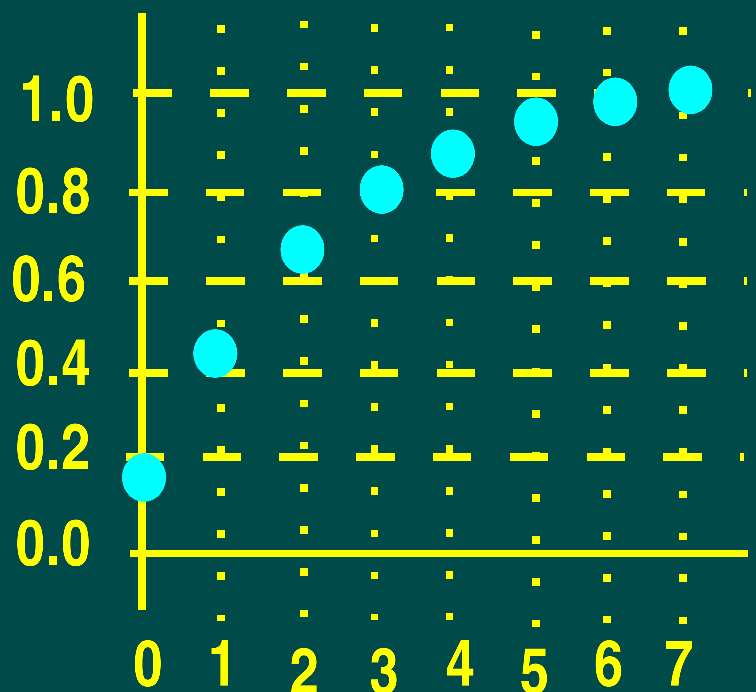
- Obtain histogram of the given image.
- Equalize the image to precompute a mapped level s_k for each r_k . T
- Obtain G from the specified histogram by equalization.
- Precompute z_k for each s_k using iterative scheme (3.3-17)
- Map r_k-> s_k and back to z_k .
- Moon example 3.4, page 100, 101, 102 GW



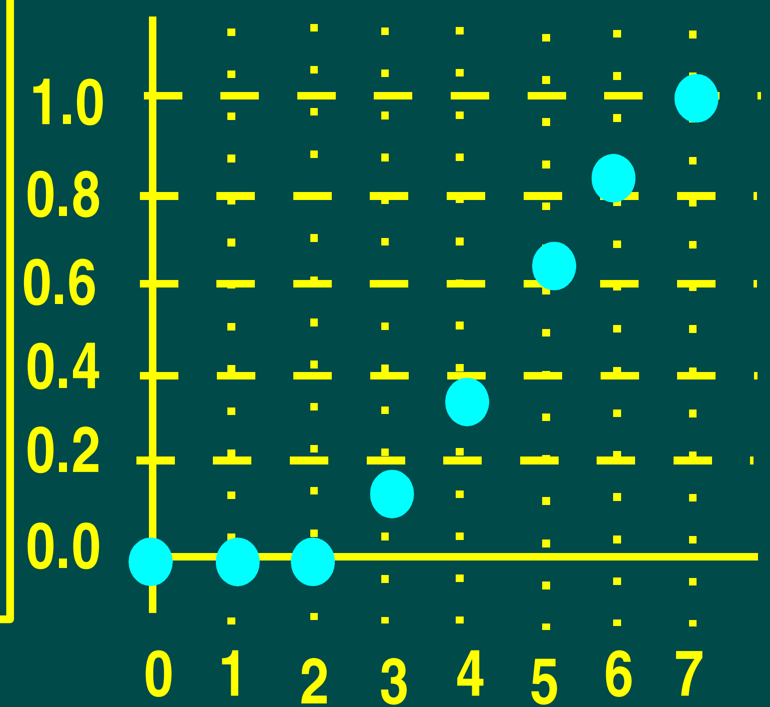
desired

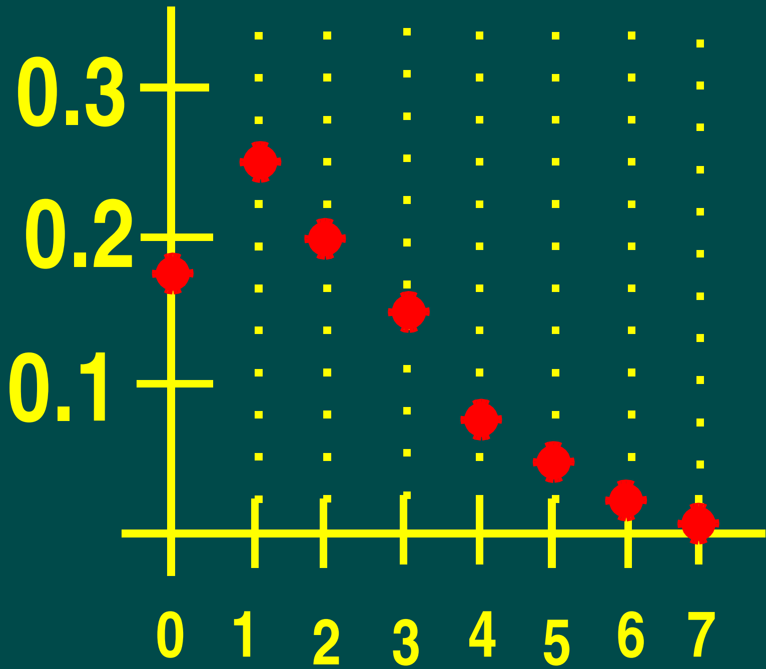


0 – 0.19
1 – 0.25
2 – 0.21
3 – 0.16
4 – 0.08
5 – 0.06
6 – 0.03
7 – 0.02

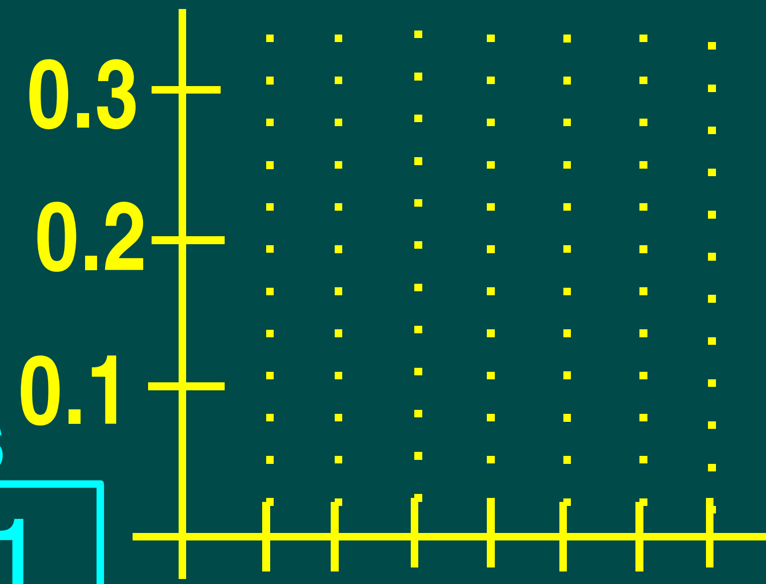


0 – 0.0
1 – 0.0
2 – 0.0
3 – 0.15
4 – 0.20
5 – 0.30
6 – 0.20
7 – 0.15

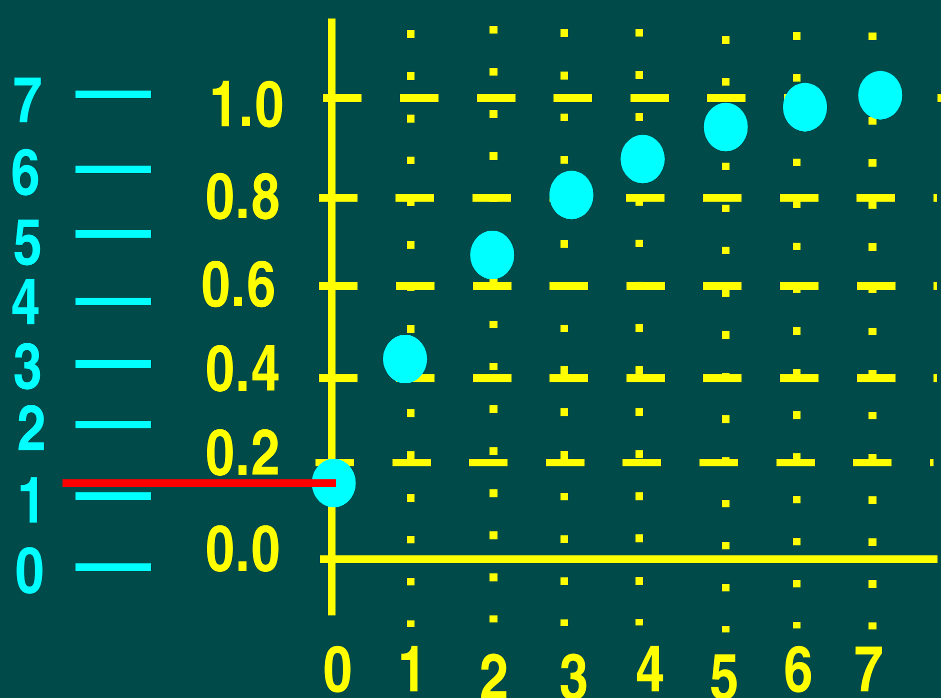




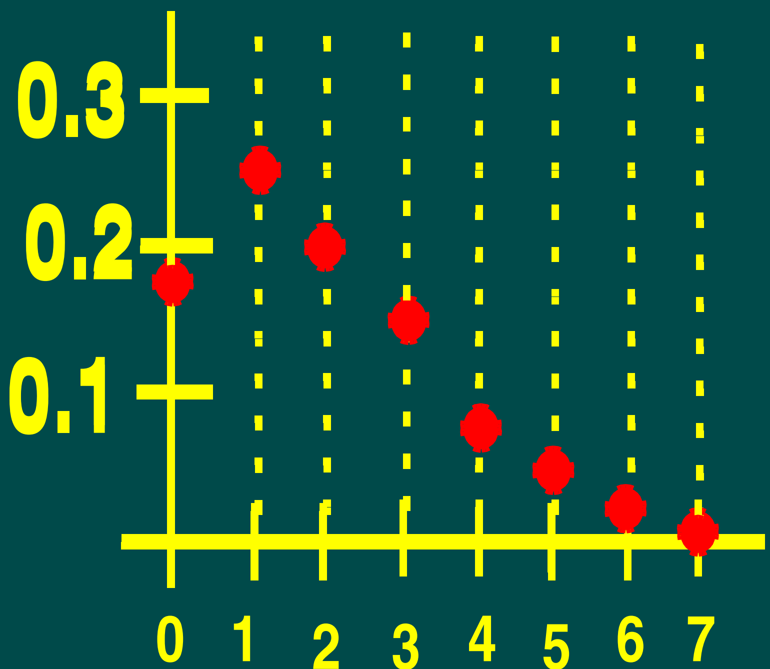
equalized histogram

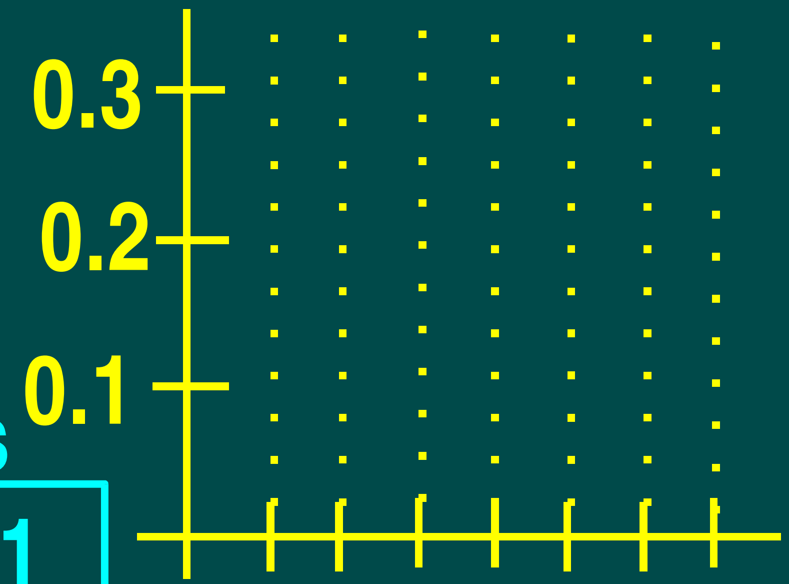


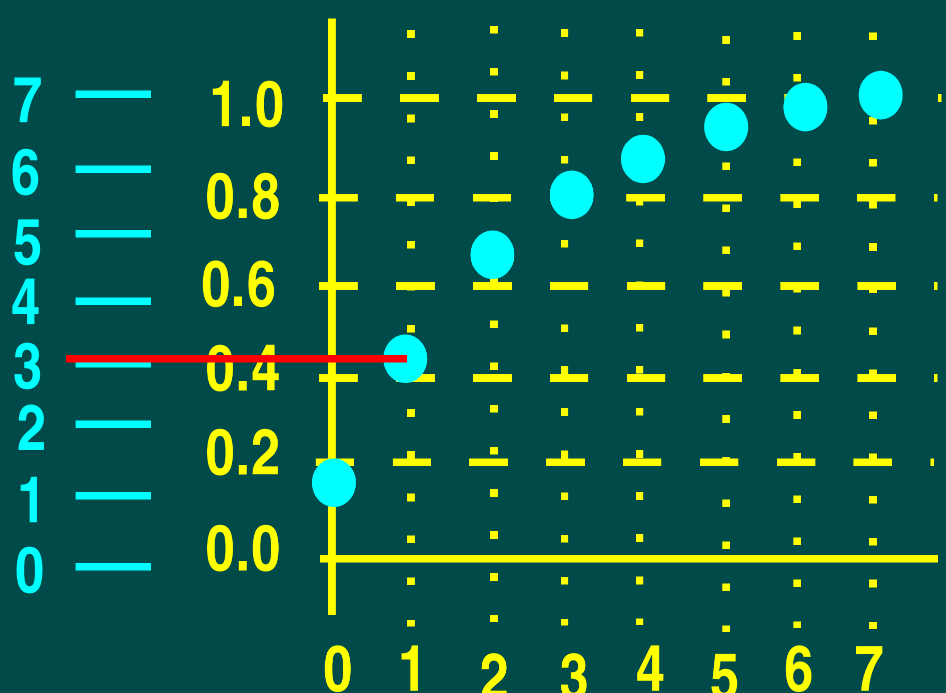
r	s
0	1



equalized histogram

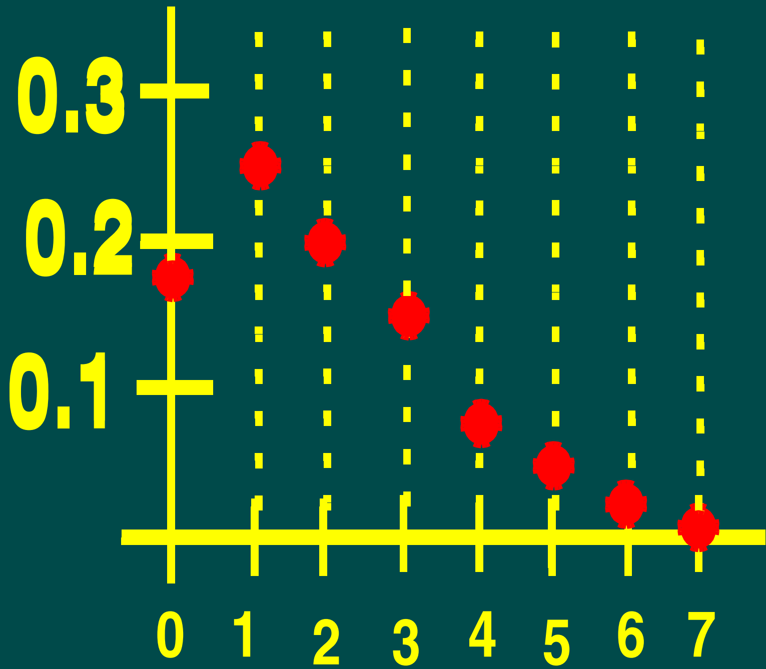


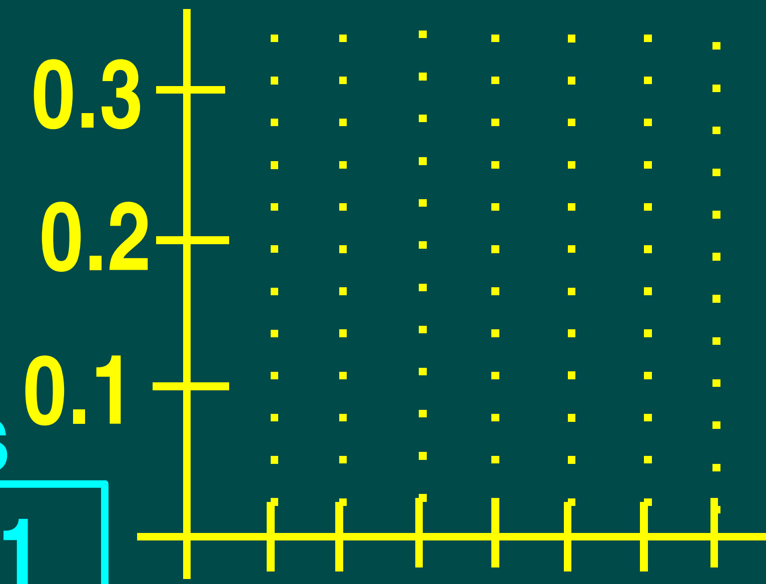




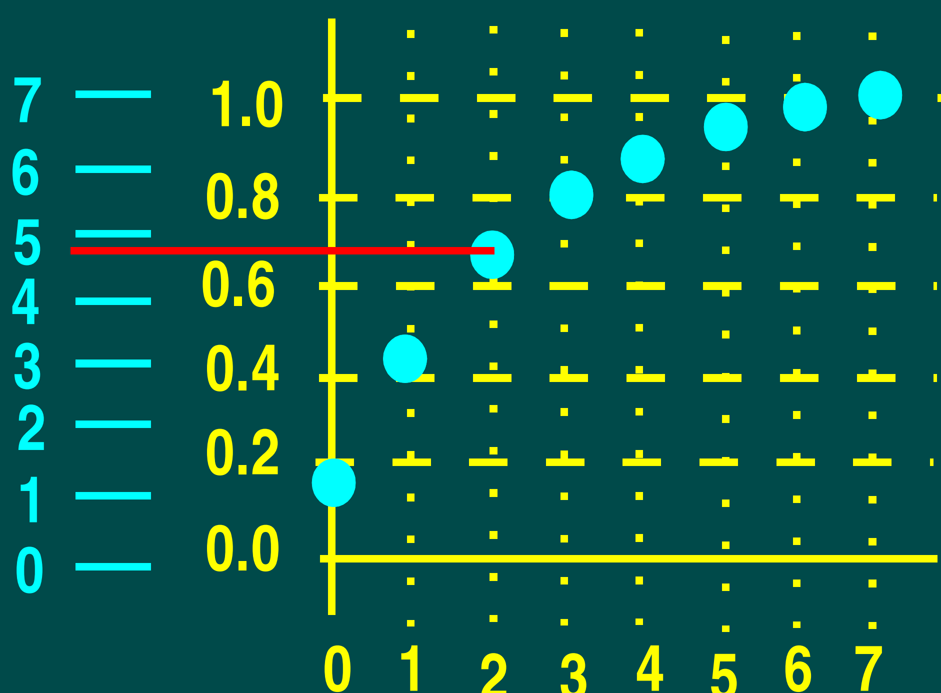
r	s
0	1
1	3

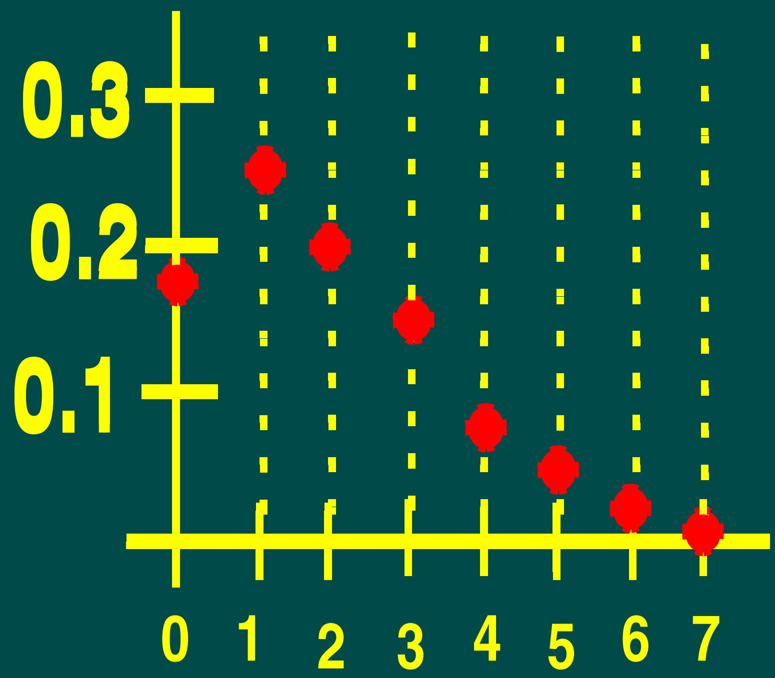
equalized histogram



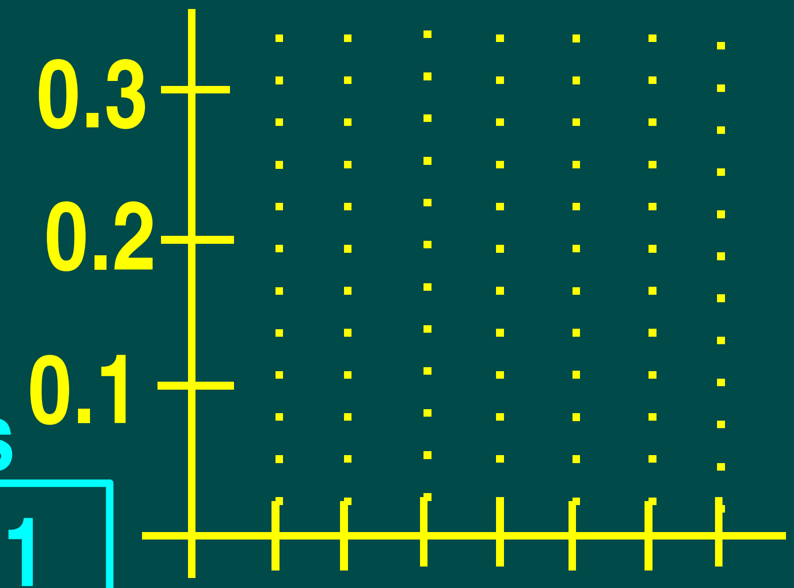


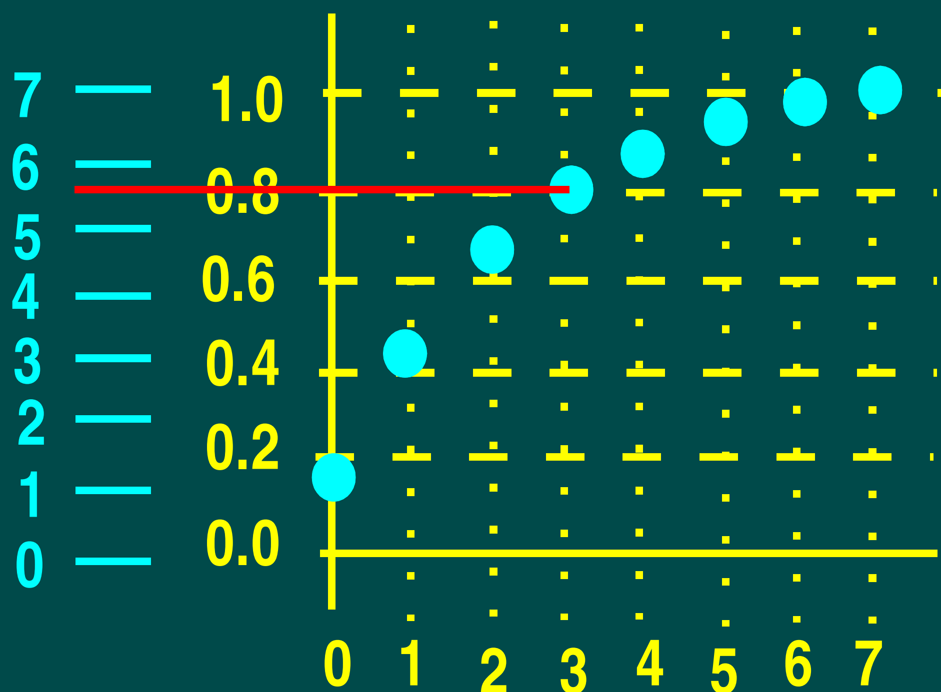
r	s
0	1
1	3
2	5





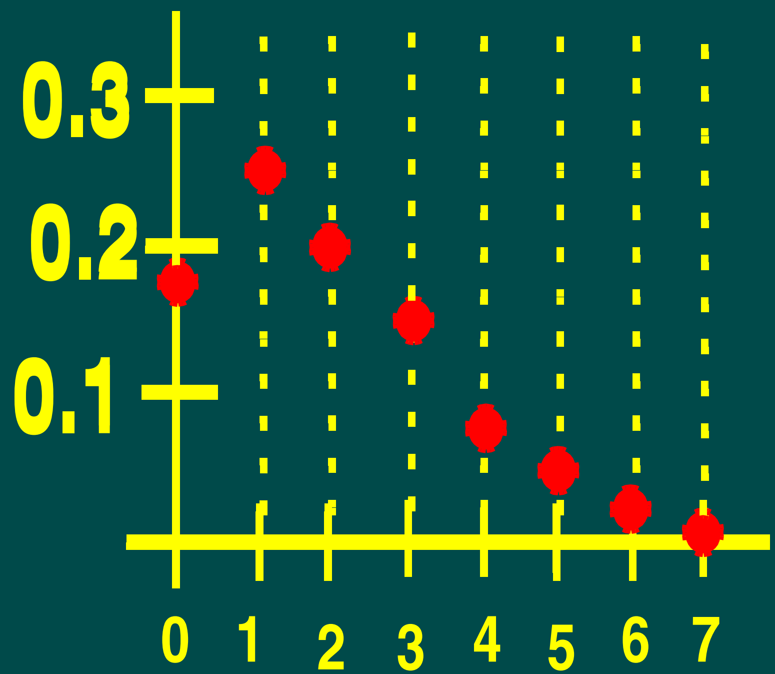
equalized histogram

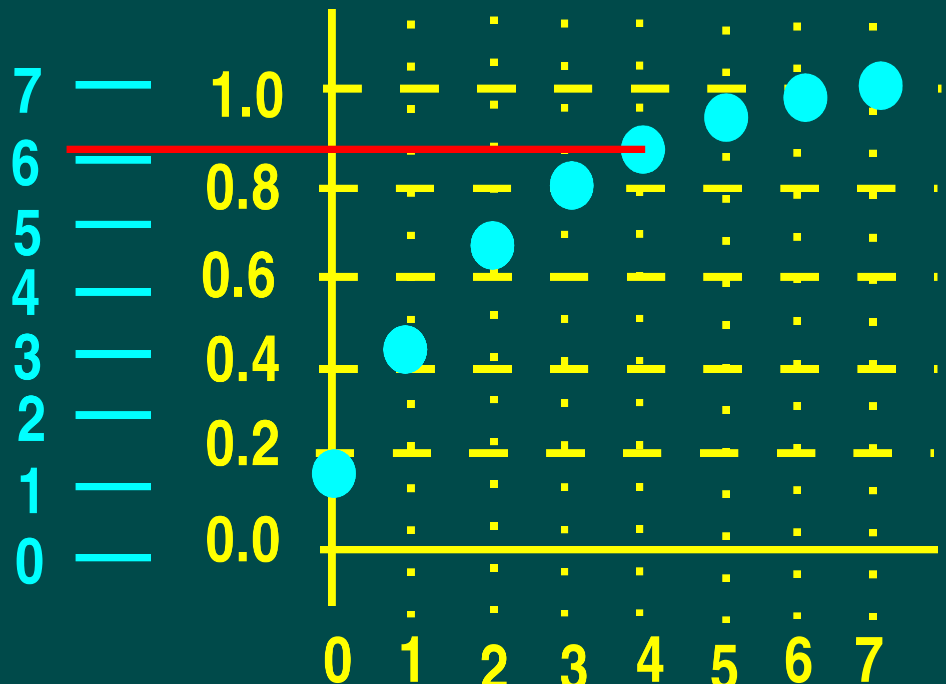




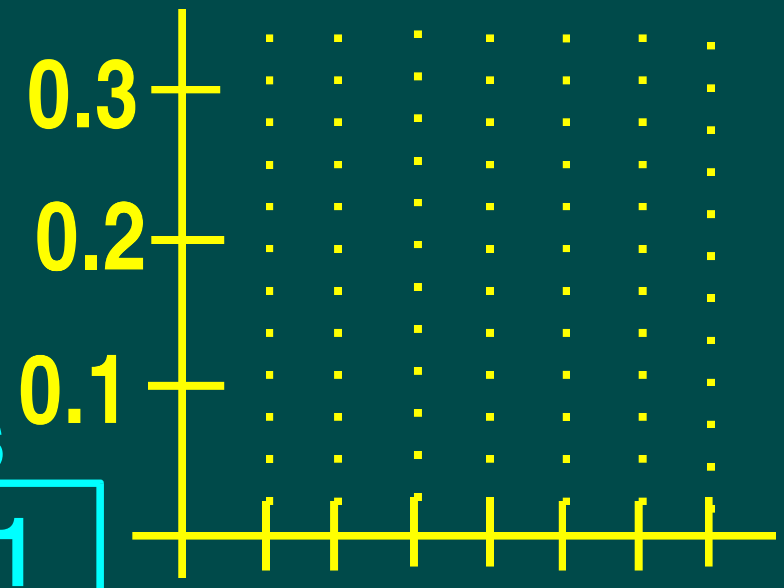
r	s
0	1
1	3
2	5
3	6

equalized histogram

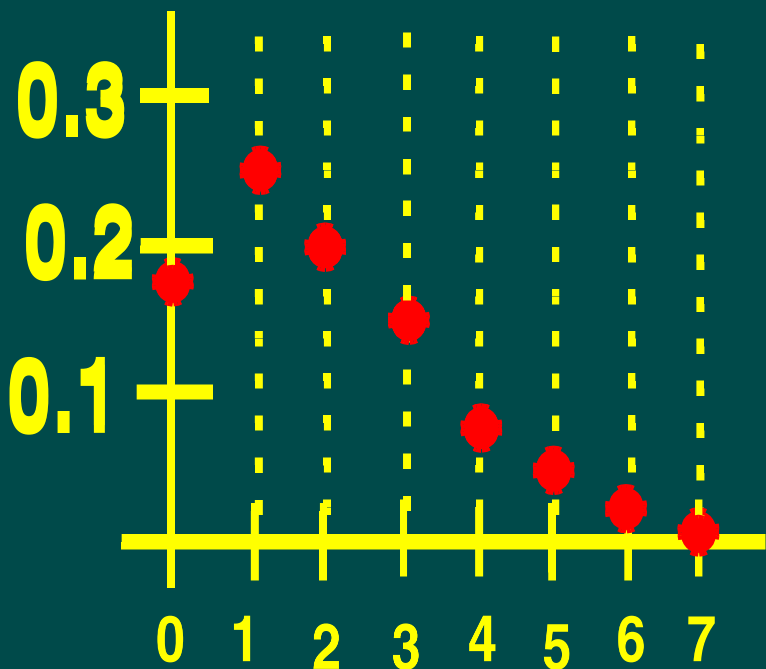


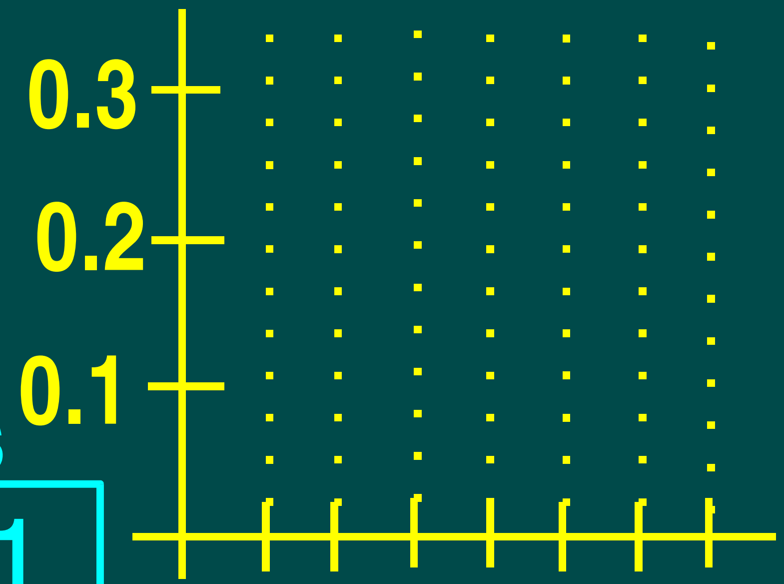


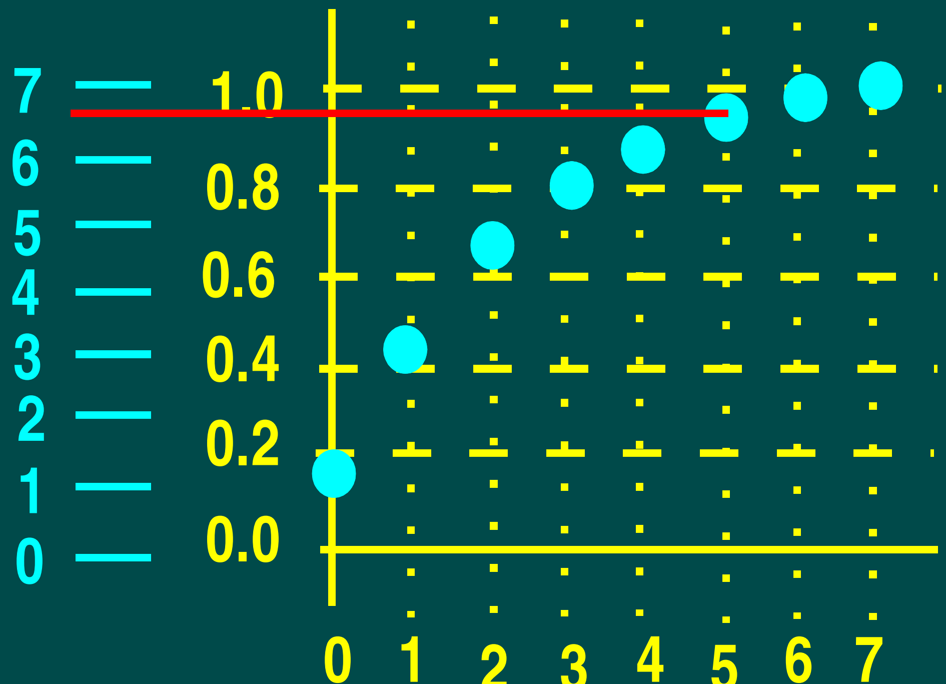
r	s
0	1
1	3
2	5
3	6
4	6



equalized histogram

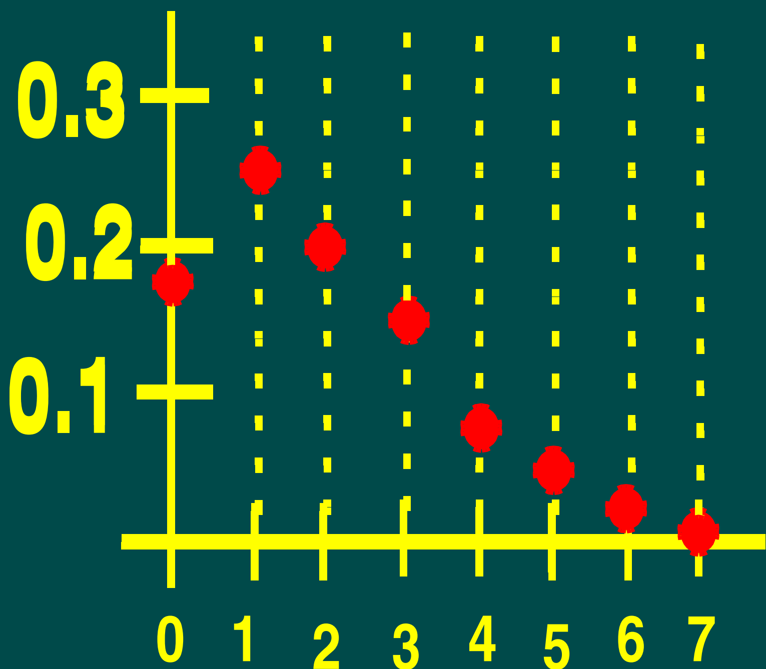


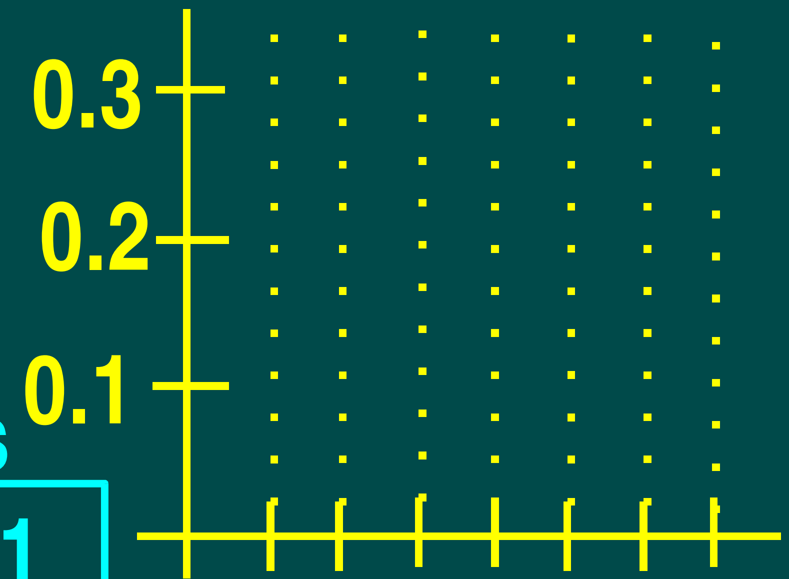




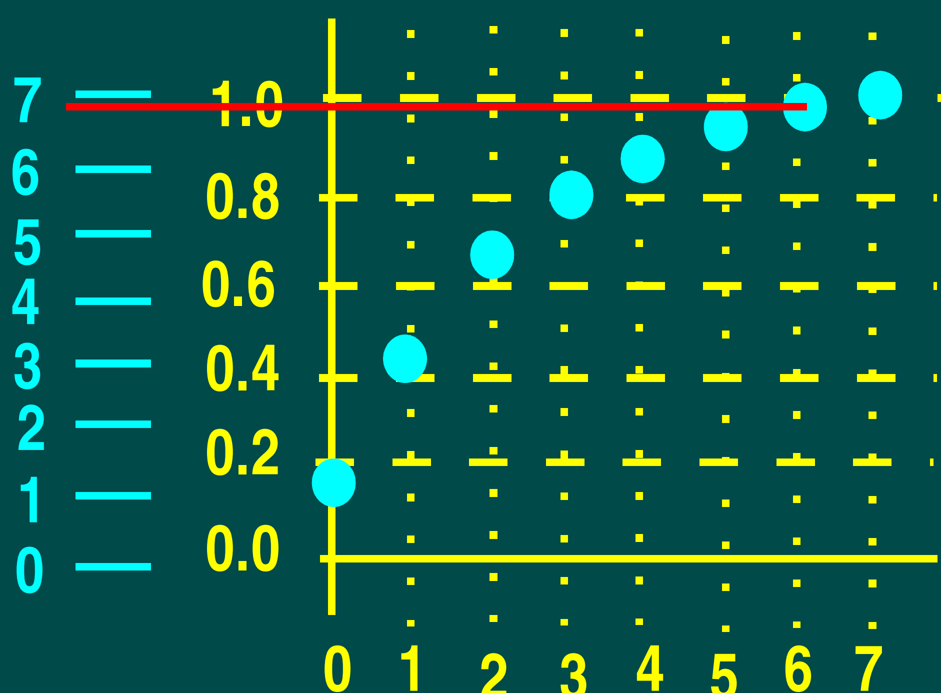
r	s
0	1
1	3
2	5
3	6
4	6
5	7

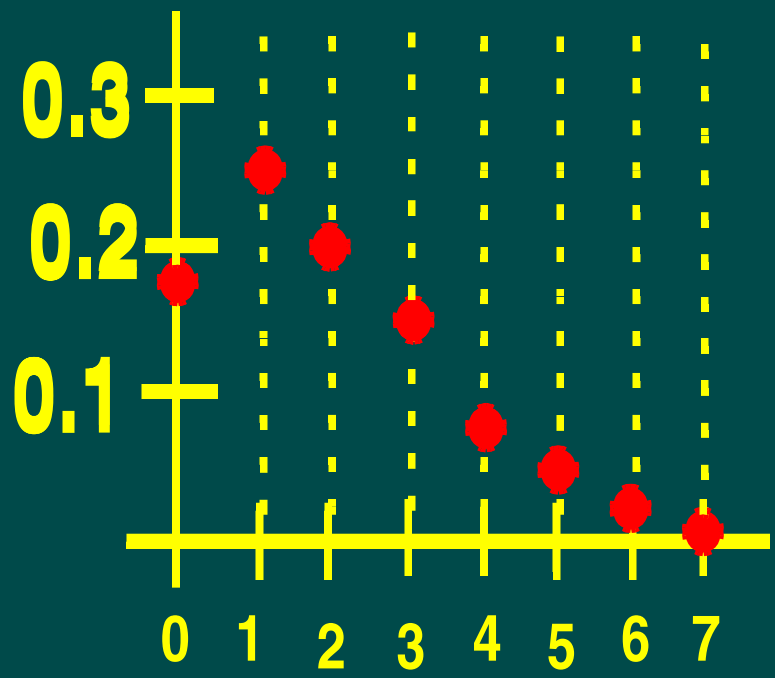
equalized histogram

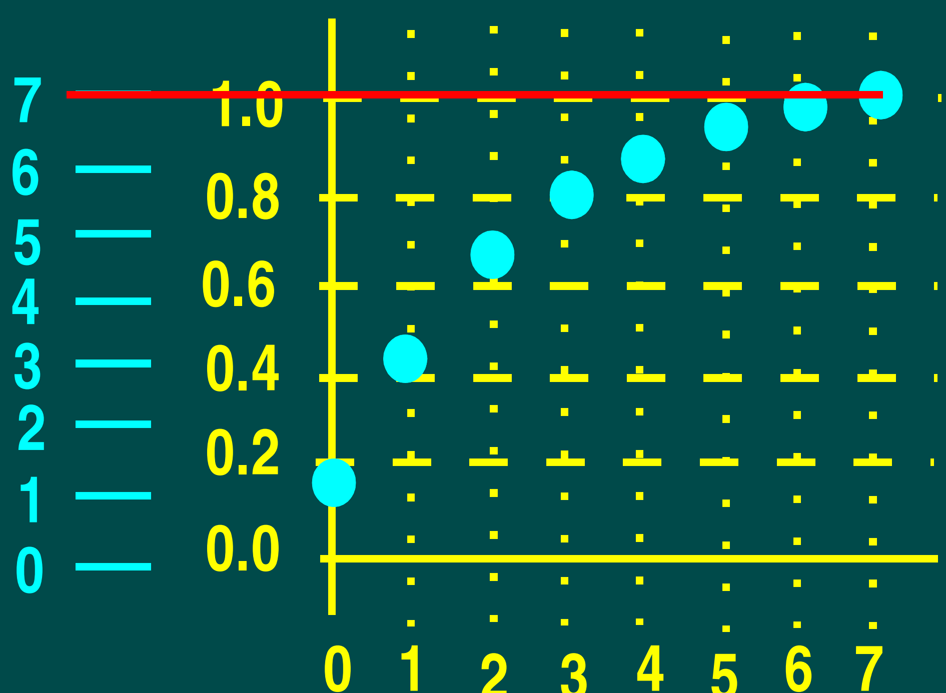




r	s
0	1
1	3
2	5
3	6
4	6
5	7
6	7

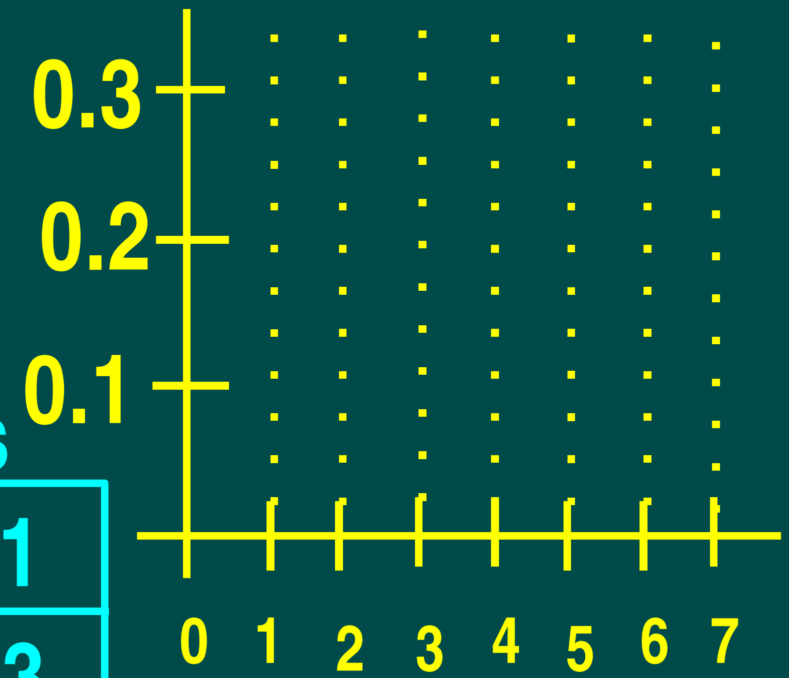


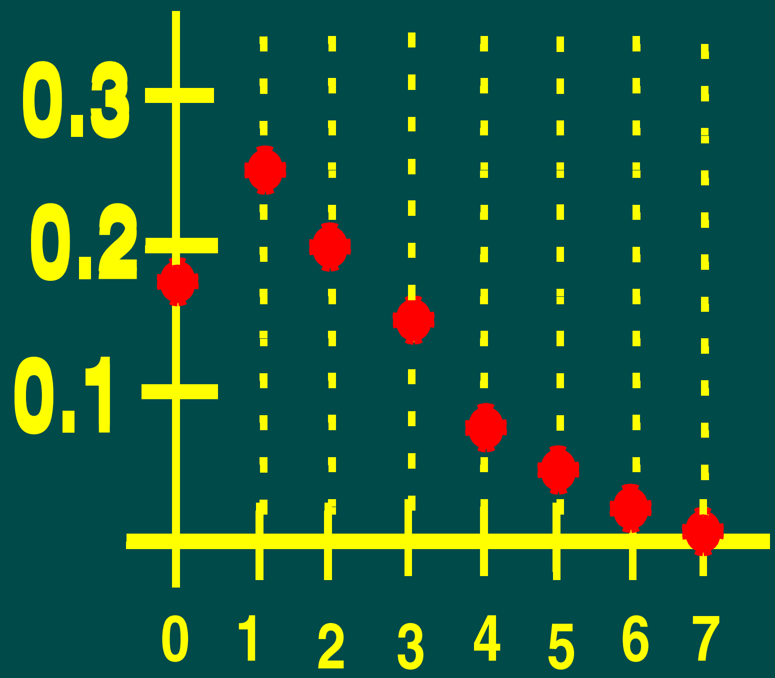




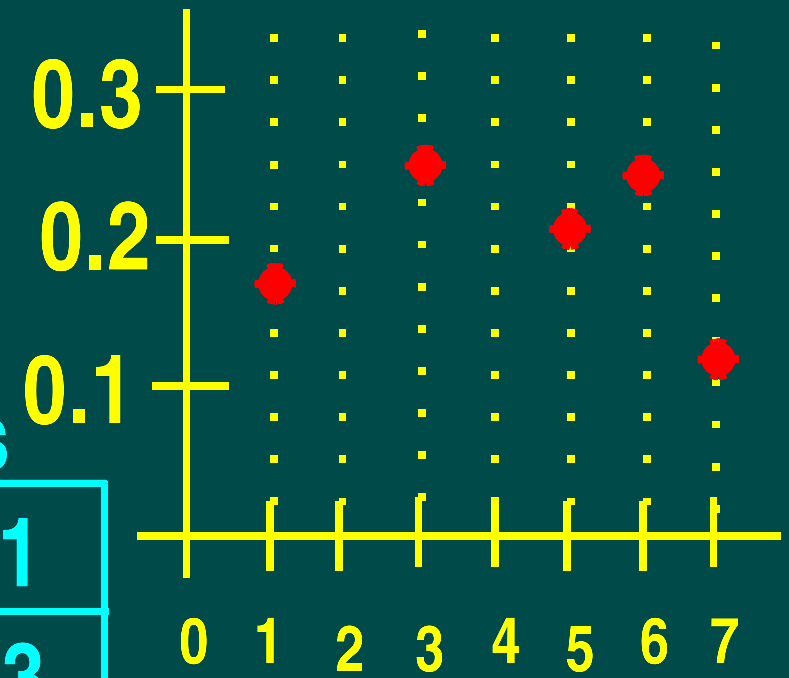
equalized histogram

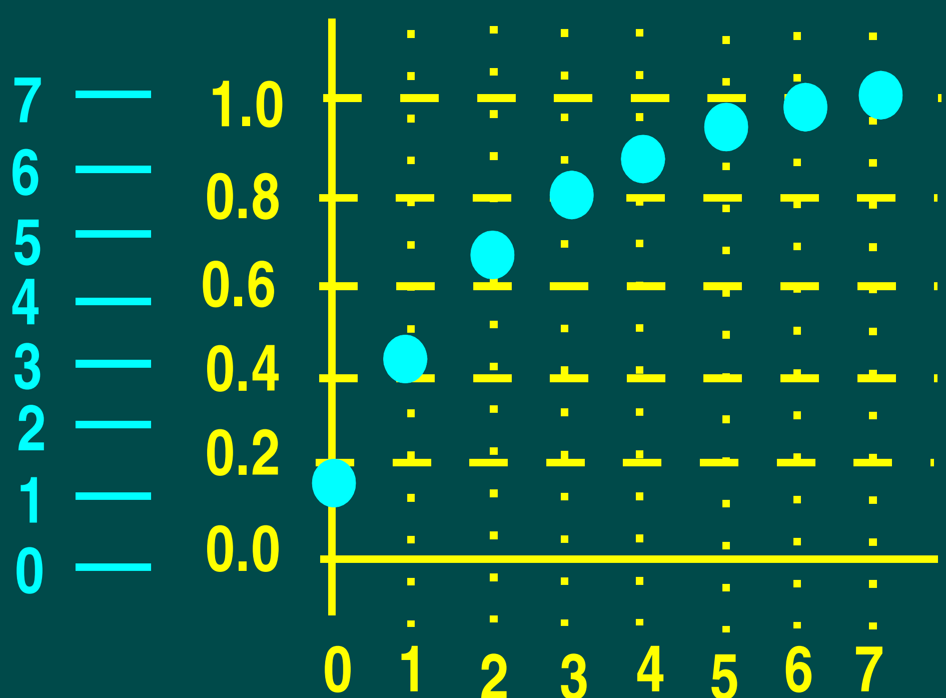
r	s
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7





equalized histogram





r	s
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7

1 = 0.19

3 = 0.25

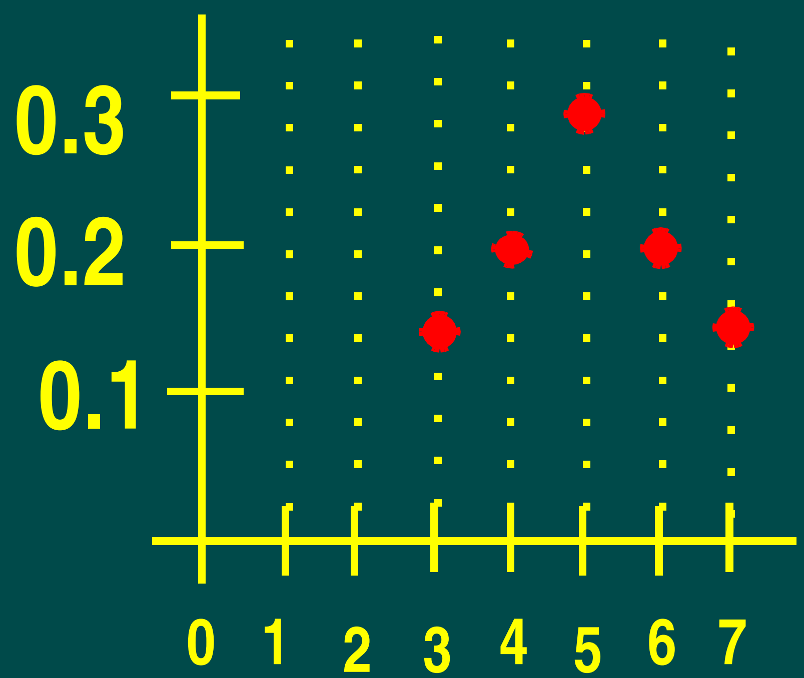
5 = 0.21

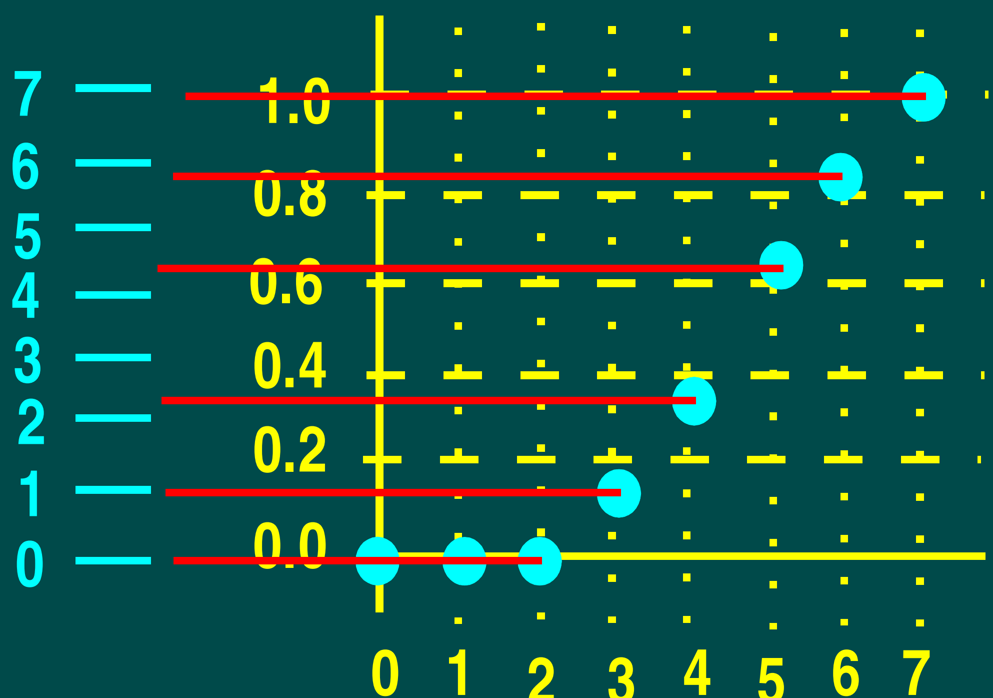
6 = .16+.08=.24

7 = .06+.03+.02

= 0.11

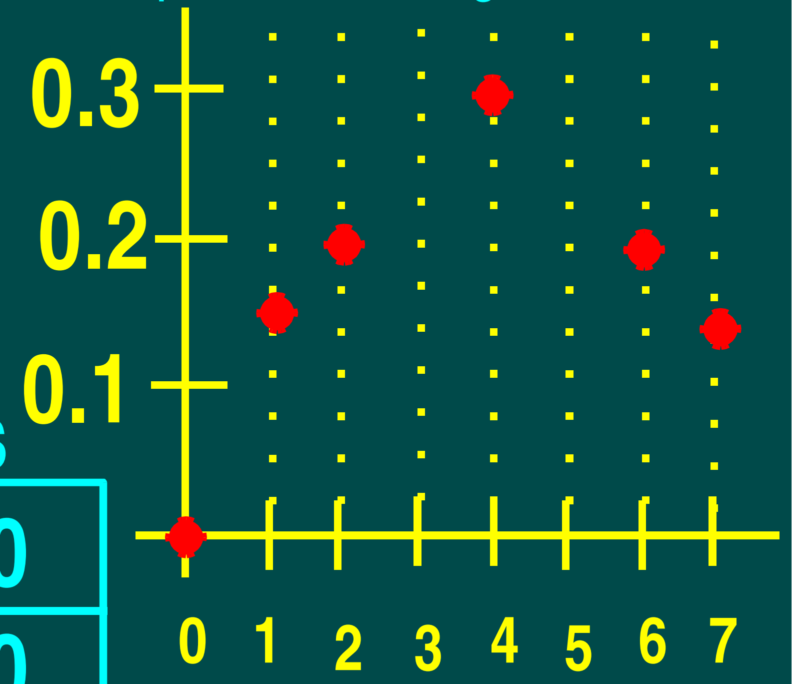
desired

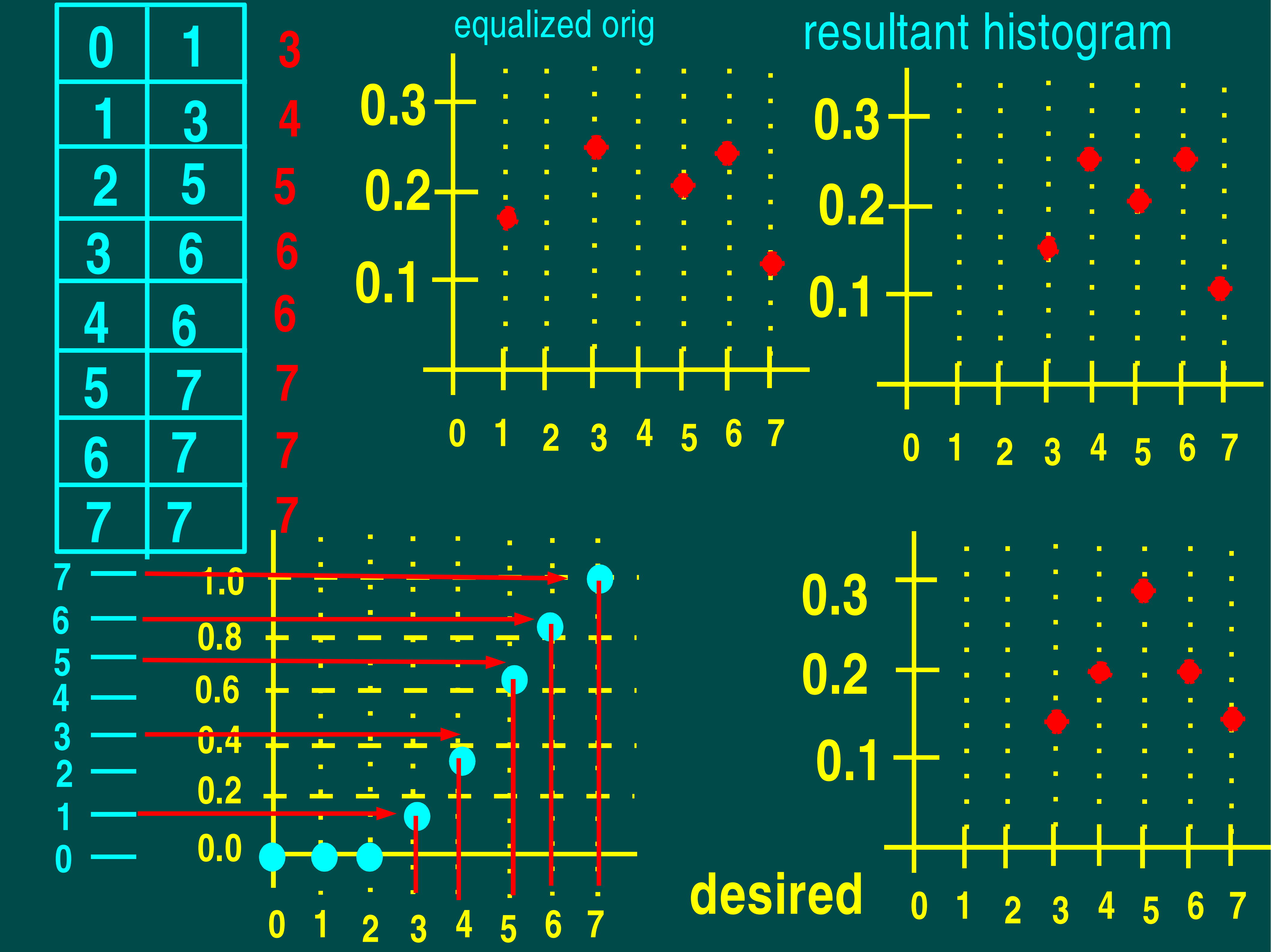




r	s
0	0
1	0
2	0
3	1
4	2
5	4
6	6
7	7

equalized histogram





equalized orig
resultant histogram
desired
0 1
1 3
2 5
3 6
4 6
5 7
6 7
7 7
3 4 5 6 6 7 7 7
0.3
0.2
0.1
0 1 2 3 4 5 6 7
1.0
0.8
0.6
0.4
0.2
0.0