Histogram Specification

equalize

desired

G equalize

G⁻¹
Histogram Specification (cont.)

- Equalize the levels of the original image.
- Specify the desired density function (histogram) and obtain the transformation function $G(z)$.
- Apply the inverse transformation function $z = G^{-1}(s)$ to the levels obtained in first step.
Saw what we want the histogram to look like and come up with a transform function that will give it to us.

Continuous random variables r & z. Pr(r) and Pz(z) denote their probability density functions. (Continuous equivalent of a histogram).

Pr(r) input image

Pz(z) desired output image (processed). This is what we'd like the processed image to have.
Histogram Specification (cont.)

-from text-

- Continuous version of histogram equalization.
- Cumulative Distribution Function

\[ s = T(r) = \int_{0}^{r} Pr(w) \, dw \]
Continuous version of histogram equalization.
Cumulative distribution function.

\[ s = T(r) = \int_{0}^{r} P_{r}(w) \, dw \]
\[ G(z) = \int_{0}^{z} P_{z}(t) \, dt = s \]
\[ \rightarrow G(z) = T(r) \]
\[ z = G^{-1}(s) = G^{-1}(T(r)) \]
Histogram Specification (cont.)

- From text -

- If you have $Pr(r)$ estimated from input image, you can obtain $T(r)$ from

$$
\int_{0}^{r} P_{r}(w) \, dw
$$

Transformation $G(z)$ can be obtained by using

$$
\int_{0}^{z} P_{z}(t) \, dt
$$

since you specified $Pz(z)$!
Histogram Specification (cont.)

-from text-

• Assume $G^{-1}$ exists and that is satisfys:
• Single-Valued, monotonically increasing
• $0 \leq G^{-1}(z) \leq 1$ for $0 \leq z \leq 1$
You can obtain an image with the specified probability density function from an input function using the following:

- obtain $T(r)$ using equilization
- obtain $G(z)$ by equalizing specified histogram
- obtain $G(z)$
- obtain the output image by applying $G$ to all pixels in input image.
So far so good (continuously speaking)

In practice it is difficult to obtain analytical expressions for $T(r)$ and $G^{-1}$

With discrete values it becomes possible to make a close approximation to the histogram.
discrete Formulas

\[ s_k = T(r_k) = \sum_{j=0}^{k} P_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}, \quad k = 0, 1, 2, \ldots, L - 1 \]

\( L \) is the number of discrete gray levels
\( n \) = total # pixels
\( n_j \) = # of pixels with gray level j

\[ v_k = G(z_k) = \sum_{i=0}^{k} p_z(z_i) = s_k, \quad k = 0, 1, 2, \ldots, L - 1 \]

\[ z_k = G^{-1}[T(r_k)], \quad k = 0, 1, 2, \ldots, L - 1 \]

\[ z_k = G^{-1}(s_k), \quad k = 0, 1, 2, \ldots, L - 1 \]
Implementation

- Each set of gray levels is a 1D array
- All mappings from r to s and s to z are simple table lookups
- Each element (e.g. s) contains 2 important pieces of information:
  - subscript k denotes the location of the element in the array
  - s denotes the value at that location
- We need to only look at integer values
Refer to Figure 3.19 GW

- (a) a hypothetical transform function given an image that was equalized.
- (b) Equalize specified histogram. $G^{-1}$ is just running the transform backwards.
- But wait a minute! Where did we get the z's???
- We have to use an iterative scheme.
Iterative Scheme

- Obtain histogram of the given image.
- Equalize the image to precompute a mapped level $s_k$ for each $r_k$.
- Obtain $G$ from the specified histogram by equalization.
- Precompute $z_k$ for each $s_k$ using iterative scheme (3.3-17).
- Map $r_k$ -> $s_k$ and back to $z_k$.
- Moon example 3.4, page 100, 101, 102 GW
equalized histogram
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equalized histogram
equalized histogram

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equalized histogram

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1 = 0.19
3 = 0.25
5 = 0.21
6 = .16+.08=.24
7 = .06+.03+.02
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**equalized orig**

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**resultant histogram**

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**desired**

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The diagram shows the original histogram, the equalized histogram, and the desired histogram. The table lists the original values for each bin.