# Exhaustive Mut. Excl. Events

If event $B$ is composed of N mutually exclusive events, $B_1, B_2, \ldots B_N$, and those events are an exhaustive set for A, (one must occur) then,

$$P(A) = \sum_{i=1}^{N} P(A|B_i) \cdot P(B_i)$$

so,

$$\begin{aligned} P(B_i|A) &= \frac{P(A, B_i)}{P(A)} \\ &= \frac{P(A|B_i)P(B_i)}{P(A)} \\ &= \frac{P(A|B_i)P(B_i)}{\sum_{i=1}^{N} P(B_i)P(A|B_i)} \end{aligned}$$

# Bayes Law

If we have *a priori* probabilities, $P(\omega_j), j = 1, m$ that are mutually exclusive, and a random variable, $x$ that has class conditioned probabilities, $p(x|\omega_j)$,

$$P(\omega_j|x) = \frac{p(x|\omega_j)P(\omega_j)}{p(x)}$$

or,

$$posterior = \frac{(conditioned\ likelihood) \times prior}{evidence}$$

# Bayes Decision Rules

Given only the *a priori* probabilities, the logical classification is to choose $\omega_j,\ j = 1, m$ such that

$$P(\omega_j) \;=\; max\,[P(\omega_1), P(\omega_2), \ldots, P(\omega_m)]$$

But given, the class-conditioned distribution of the random variable representing the evidence $p(\mathbf{x}|\omega_j),\ j = 1, m$. We should be able to do better by choosing $\omega_j$ such that:

$$P(\omega_j|\mathbf{x}) \;=\; max\,[P(\omega_1|\mathbf{x}), P(\omega_2|\mathbf{x}), \ldots, P(\omega_m|\mathbf{x})]$$

The above is called Bayes Decision Rule

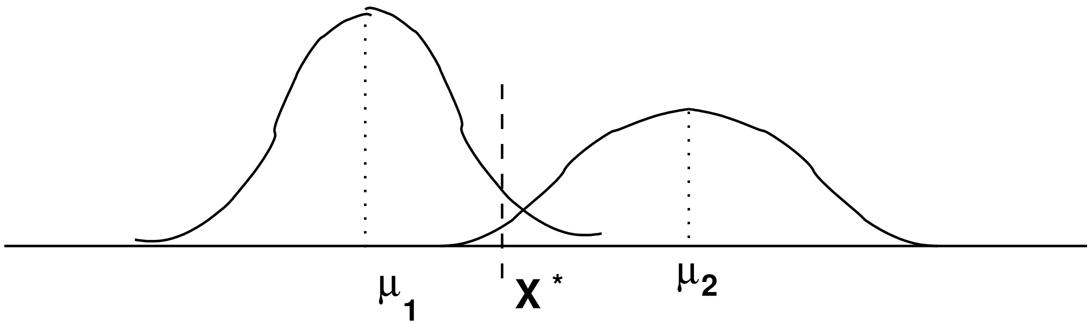
# An Example

An archeologist finds a skull in an area where two types of humanoid skulls have been found before, Homo Destructus and Home Duplicitous. Homo Destructus is found 60% of the time, so $\omega_1 = 0.60$ and $\omega_2 = 0.40$. Past studies of the skulls indicate that the distribution of skull diameters is Normal with

$\mu_1 = 165\, mm$ and $\sigma_1^2 = 28.0$
and $\mu_2 = 180\, mm$ and $\sigma_2^2 = 35.0$

The archeologist measures the skull and gets a value of 170 mm. Which population should he assign the skull to?

# *An Example Continued*




The likelihood that a value of 170mm occurs in:
**Homo Destructus ($\omega_1$)**

$$
\begin{aligned}
p(\omega_1|170) &= \frac{p(170|\omega_1)P(\omega_1)}{P(170)} \\
&= \frac{p(170|\omega_1)P(\omega_1)}{p(170|\omega_1)P(\omega_1 \; + \; p(170|\omega_2)P(\omega_2)} \\
&= \frac{0.048 \cdot 0.60}{0.048 \cdot 0.60 \; + \; 0.016 \cdot 0.40} \\
&= 0.82
\end{aligned}
$$

# An Aside

$$
\begin{aligned}
p(170|\omega_1) &= p(169.5 < x < 170.5) \\
&= p(\frac{169.5 - 165}{\sqrt{28}} < z < p(\frac{170.5 - 165}{\sqrt{28}}) \\
&= p(0.85 < z < 1.04)
\end{aligned}
$$

From a table of Normal values (0-x area)

$$
\begin{aligned}
p(z < 0.85) &= 0.30234 \\
p(z < 1.04) &= 0.35083
\end{aligned}
$$

$$p(170|\omega_1) = 0.35083 - 0.30234 = 0.0485$$

# An Example Continued

**Homo Duplicitous ($\omega_2$)**

$$\begin{aligned} &= \frac{p(170|\omega_2)P(\omega_2)}{p(170|\omega_1)P(\omega_1 \; + \; p(170|\omega_2)P(\omega_2} \\ &= \frac{0.016 \cdot 0.40}{0.048 \cdot 0.60 \; + \; 0.016 \cdot 0.40} \\ &= 0.18 \end{aligned}$$

So class 1, Homo Destructus is the best choice.

# The Critical Value

We can find the point where $p(x|\omega_1) = p(x|\omega_2)$, x*, so that the classifier becomes:

$$Choose\, \omega_1\ if\ x < x*,\ otherwise,\ choose\, \omega_2.$$

| x | $\omega_1$\|x) | $\omega_2$\|x) |
|---|---|---|
| 170 | 0.82 | 0.18 |
| 172 | 0.69 | 0.39 |
| 173 | 0.51 | 0.49 |
| 173.1 | 0.50 | 0.50 |
| 174 | 0.40 | 0.60 |

Bayes Decision Rule for two classes is:

Choose $\omega_1$ if $P(\omega_1|x) > P(\omega_2|x)$; otherwise choose $\omega_2$.

So, Choose $\omega_1$ if x < 173.1, otherwise, choose $\omega_2$.