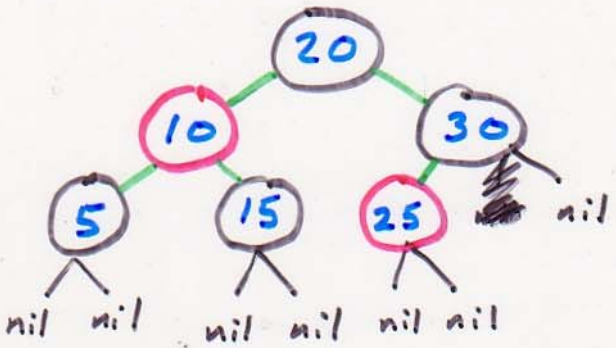
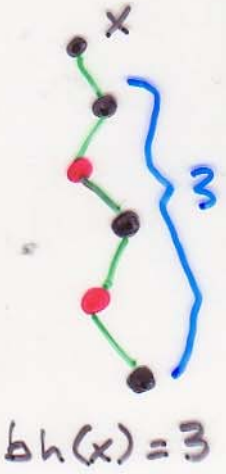
CS 223 Lec 10



- this tree meets all requirements, so <u>is</u> a Red-Black tree.

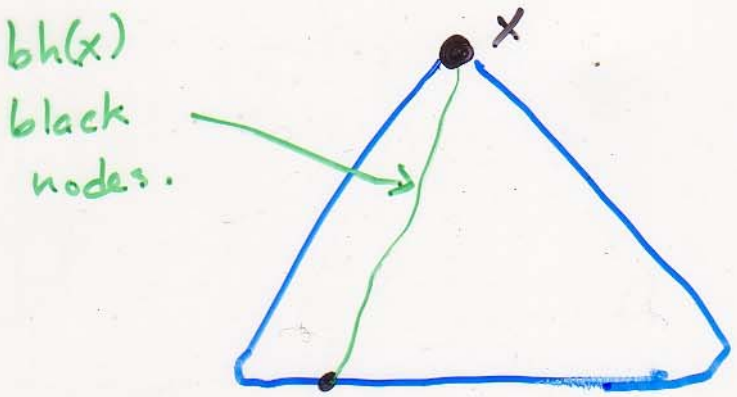
Height of R-B Trees

Defn the black-height (bh) of a node x is the # of black nodes on any path from x to a descendent leaf.



Claim Any node x, the number of internal nodes in the subtree rooted at x is at least

$$2^{bh(x)} - 1$$



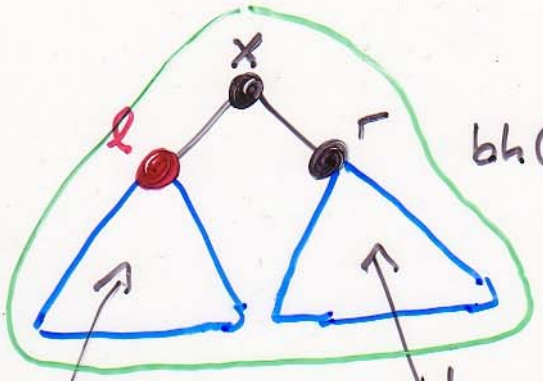
Pf (induction) on ~~b~~h of x

Base Case ~~b~~$h(x) = 0$ x is a leaf

So subtree(x) contains 0 internal nodes.

and $2^{b-h(x)} - 1 = 2^0 - 1 = 0$.

Inductive - Step $h(x) = k > 0$



$bh(r) = bh(x) - 1$

$bh(l) = bh(x)$

Since l could be black same bound applies.

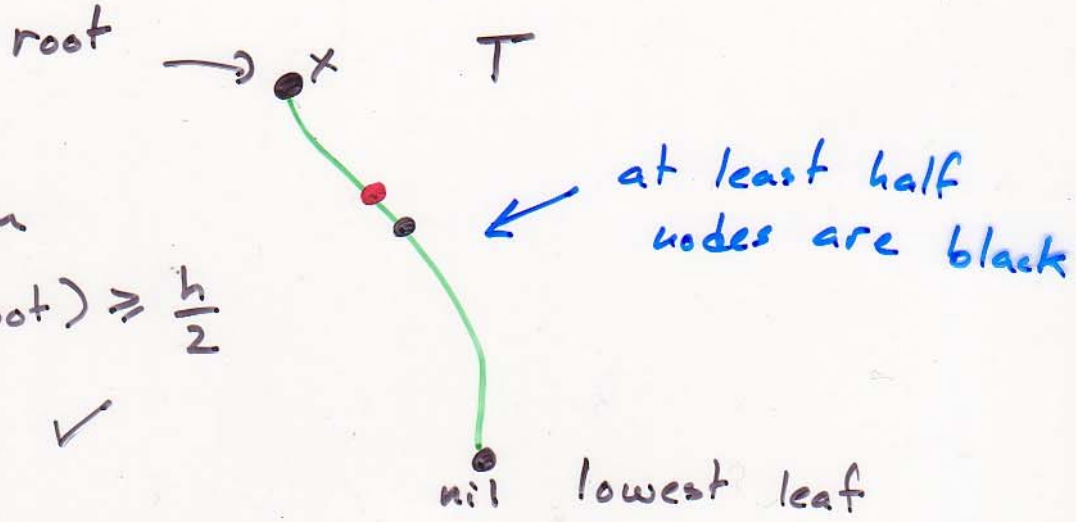
by induction hypothesis $\geq 2^{bh(x)-1} - 1$

Inside subtree(x) we have

$$\geq 1 + 2 \cdot (2^{bh(x)-1} - 1)$$

$$= 2^{bh(x)} - 1 \text{ internal nodes}$$

Suppose we red-black tree T of height h.



Claim

$b\text{-}h(root) \geq \frac{h}{2}$ ✓

# of elements

We just showed

n = # of int. nodes in subtree(x)

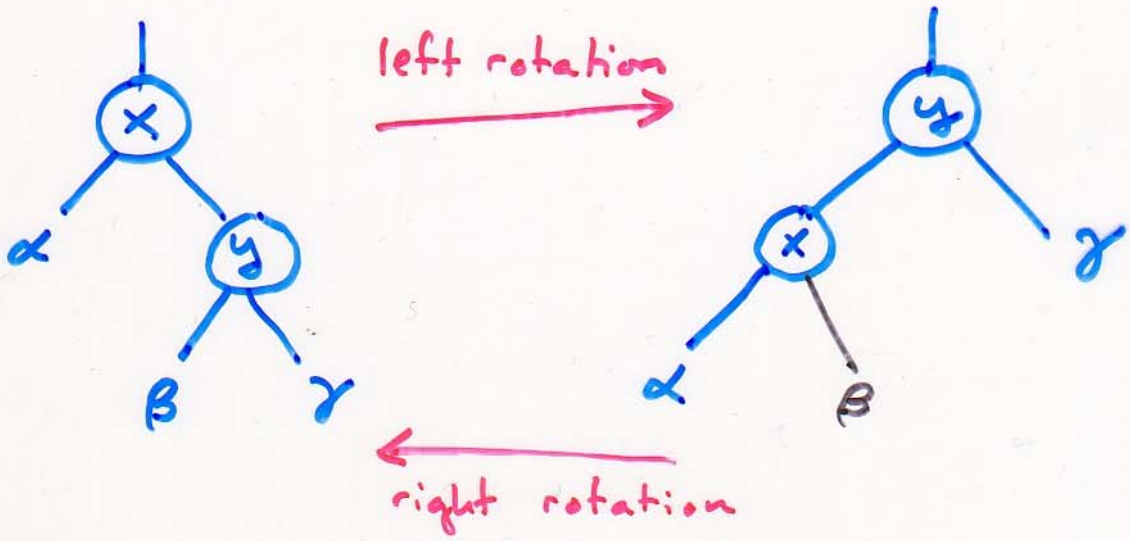
$$\geq 2^{bh(x)} - 1$$

$$n+1 \geq 2^{bh(x)}$$

$$\Rightarrow \lg(n+1) \geq bh(x) \geq h/2$$

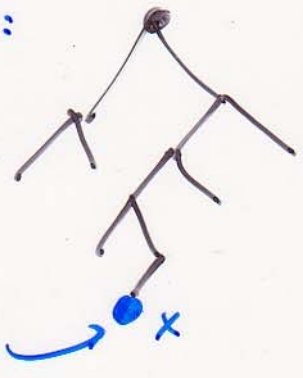
$$\Rightarrow \boxed{h \leq 2 \cdot \lg(n+1)} \; *$$

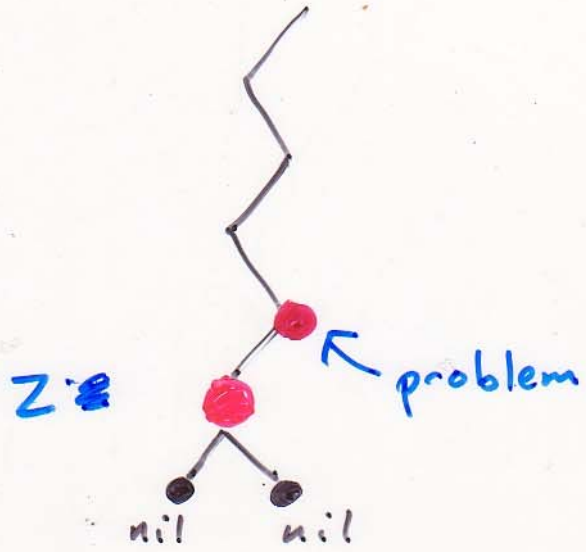
Tree Rotations



Inserting a new node in a R-B tree..

x ●:




problem
nil
nil