CS 223

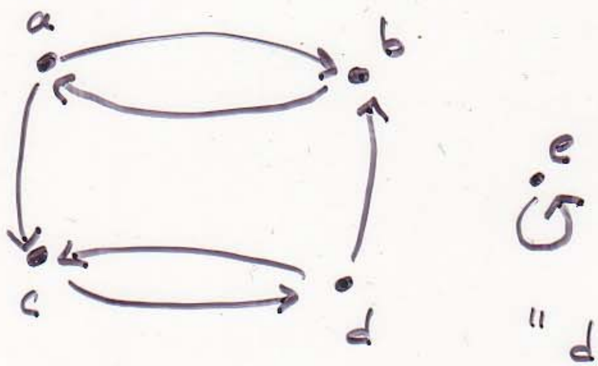
Graph Representation

adjacency matrix

graphs $G = (V, E)$

vertices edges

(directed / undirected
weighted
multigraph u v)



"disconnected"

space
$O(|V|^2)$

$$\begin{array}{c|ccccc|} & a & b & c & d & e \\ \hline a & & 1 & 1 & & \\ b & 1 & & & 1 & \\ c & & & & 1 & \\ d & & 1 & 1 & & \\ e & & & & & 1 \\ \hline \end{array}$$

"0" elsewhere

if storing an undirected graph, just store half the matrix (since symmetric)

adjacency list

$\underline{V}$

$a \rightarrow b, c$
$b \rightarrow a$
$c \rightarrow d$
$d \rightarrow b, c$
$e \rightarrow e$

space used
$O(|E|)$

"Square of a Graph"

given: $G = (V, E)$

adj. matrix M

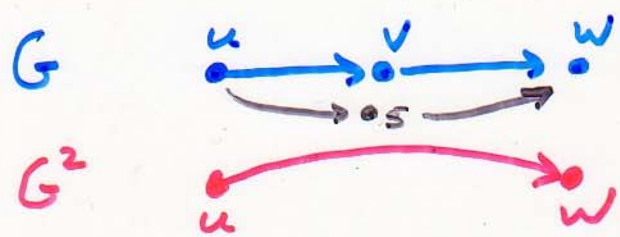
<u>Define</u>: the "square of G"

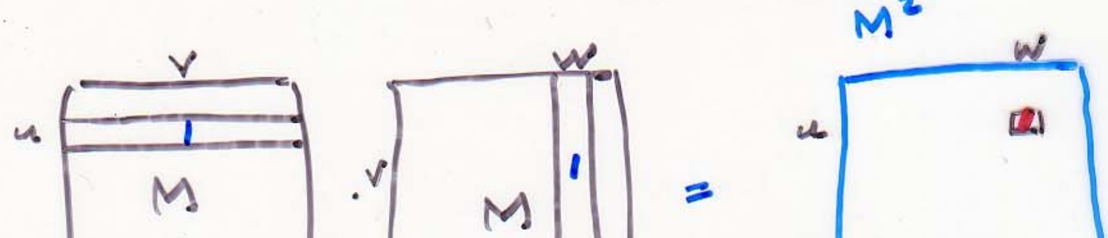
as $G^2 = (V, E^2)$

$(u, w) \in E^2$

$\Longleftrightarrow \exists v \in V$

$(u, v), (v, w) \in E$



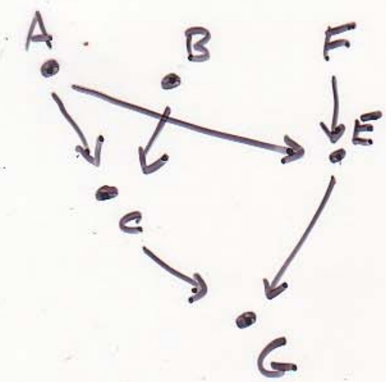


$$(u, w) \in E^2 \Longleftrightarrow \underbrace{M^2(u, w)}_{\substack{\| \\ \#\text{ of length 2 paths from} \\ u \text{ to } w \text{ in } G^2}} > 0$$

Graph Algorithms

- minimum spanning tree

- shortest-path problems
- topological sort:

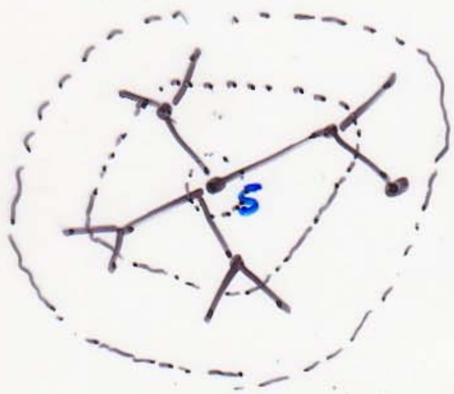


"Searching a Graph"

1. Breadth-First Search
2. Depth-First Search

Breadth First Search

given : - $G = (V, E)$
(in some representation)
- a source vertex s



Vertex Coloring scheme

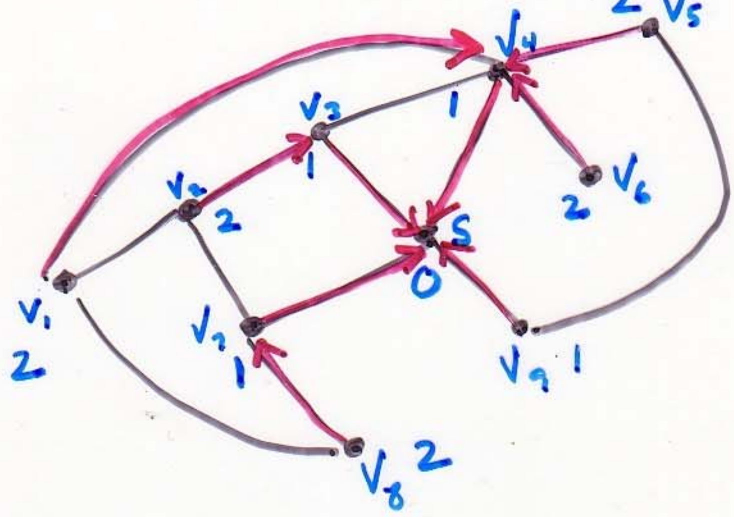
color change of any vertex

white - unvisited
grey - discovered, unfinished
black - finished

$d[u]$ = (at end of search) shortest path distance from s to u.

BFS example

	Color
s	$\cancel{G}$ B
v_1	$\cancel{W}$ $\cancel{G}$ B
v_2	$\cancel{W}$ $\cancel{G}$ B
v_3	$\cancel{W}$ $\cancel{G}$ B
v_4	$\cancel{W}$ $\cancel{G}$ B
v_5	$\cancel{W}$ $\cancel{G}$ B
v_6	$\cancel{W}$ $\cancel{G}$ B
v_7	$\cancel{W}$ $\cancel{G}$ B
v_8	$\cancel{W}$ $\cancel{G}$ B
v_9	$\cancel{W}$ $\cancel{G}$ B



$Q = \{\cancel{s}\}$

$\{\cancel{v_3}, \cancel{v_4}, v_7, v_9, \cancel{v_2}\}$

$\{\cancel{v_7}, \cancel{v_9}, \cancel{v_2}, \cancel{v_5}, \cancel{v_6}, \cancel{v_1}, \cancel{v_8}\}$