CS 350 Theory of Computation

Assignment 1 (8 marks)

Question 1 (1 marks)

Prove that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.

Question 2 (1 marks)

Given a convex polygon $C(n)$ with n vertices, prove that one can always decompose $C(n)$ into $n - 2$ triangles using diagonals (a diagonal is a line segment connecting two vertices of $C(n)$).

Question 3 (2 marks)

Given a graph $G = (V, E)$, we define $w(G)$ as the number of connected components in G. A *cut edge* of G is an edge $e \in E$ such that $w(G - \{e\}) > w(G)$. $(G - \{e\} = (V, E - \{e\}).)$

Prove that $e \in E$ is a cut edge of G if and only if e is contained in no cycle of G.

Question 4 (2 marks)

A fully binary tree T is a tree such that all internal nodes have two children. Prove that a fully binary tree with n internal nodes has $n + 1$ leaves (a leave is a node with no child).

Question 5 (2 marks)

Given an undirected graph $G = (V, E)$, the breadth-first-search starting at $v \in V$ ($bfs(v)$ for short) is to generate a shortest path tree starting at vertex $v \in V$. The diameter of G is the longest of all shortest paths $\delta(u, v), u, v \in V$.

When G is a tree, the following algorithm is proposed to compute the diameter of G.

1. Run $bfs(w), w \in V$ and compute the vertex $x \in V$ furthest from w.
2. Run $bfs(x)$ and compute the vertex $y \in V$ furthest from x.
3. Return $\delta(x, y)$ as the diameter of G.

Prove that this algorithm is correct; i.e., $\delta(x, y)$ is in fact the longest among all the shortest paths between $u, v \in V$.

Date Due: before the end of class on **Monday, January 30, 2006.** Late assignment will lose 2 marks for each overdue day.