CS 350 Theory of Computation

Assignment 1 (8 marks)

Question 1 (2 marks)

Given an undirected graph $G = (V, E)$, the breadth-first-search starting at $v \in V$ ($bfs(v)$ for short) is to generate a shortest path tree starting at vertex $v \in V$. The diameter of G is the longest of all shortest paths $\delta(u, v), u, v \in V$.

When G is a tree, the following algorithm is proposed to compute the diameter of G.

1. Run $bfs(w), w \in V$ and compute the vertex $x \in V$ furthest from w.
2. Run $bfs(x)$ and compute the vertex $y \in V$ furthest from x.
3. Return $\delta(x, y)$ as the diameter of G.

Prove that this algorithm is correct; i.e., $\delta(x, y)$ is in fact the longest among all the shortest paths between $u, v \in V$.

Question 2 (2 marks)

Given a convex polygon $C(n)$ with n vertices, prove that one can always decompose $C(n)$ into triangles using $n - 3$ diagonals (a diagonal is a line segment connecting two vertices of $C(n)$).

Question 3 (2 marks)

Show that in any simple graph there is a path from any vertex of odd degree to some other vertex of odd degree.

Question 4 (2 marks)

A fully binary tree T is a tree such that all internal nodes have two children. Prove that a fully binary tree with n internal nodes in total has $2n + 1$ nodes.

Date Due: before the end of class on **Thursday, February 3, 2005.** Late assignment will lose 2 marks for each overdue day.