H/W 3 Solns

7b/ $\underline{a \mid b,\ a \mid c \Rightarrow a^2 \mid b \cdot c}$

$b = d_1 a$ (since $a \mid b$)

$c = d_2 a$ (since $a \mid c$)

$$b \cdot c = (d_1 a)(d_2 a)$$
$$= d_1 d_2 \cdot a^2$$

So $a^2 \mid b \cdot c$

13 b/ $b_n = 3 b_{n-1} + 2 b_{n-2}$

i. $3 \mid b_{n-2} \Rightarrow 3 \mid b_n$

$b_{n-2} = 3 k_n$

$$b_n = 3 b_{n-1} + 2(3 k_n)$$
$$= 3(b_{n-1} + 2 k_n)$$

$\Rightarrow 3 \mid b_n$

19/ "$5 \mid n^2 \Rightarrow 5 \mid n$"

$\equiv$ ($p \to q \equiv \neg q \to \neg p$

contrapositive: $5 \nmid n \Rightarrow 5 \nmid n^2$

$\uparrow$ = doesn't divide

$5 \nmid n \Rightarrow$

$$n = 5k + r \quad (r = 1, 2, 3, 4)$$

$$\begin{aligned} n^2 &= (5k + r)^2 \\ &= 25k^2 + 10kr + r^2 \\ &= 5(5k^2 + 2kr) + r^2 \end{aligned}$$

Since $r^2 \in \{1, 4, 9, 16\}$

n^2 is <u>not</u> divisible by 5.

4c/ $b_1 = 2$

$b_k = 3b_{k-1} + 2$ for $k \geq 2$

Prove $\boxed{b_n = 3^n - 1}$ $S(n)$

<u>Proof</u> by induction

<u>Step 1 (basis)</u> $n = 1$

$b_1 = 2$ $\quad b_1 = 3^1 - 1 = 2$ ✓

<u>Step 2 (ind. step)</u>

Assume $\boxed{b_{k-1} = 3^{k-1} - 1}$ $S(k-1)$

for $n = k$,

$$
\begin{aligned}
b_k &= 3b_{k-1} + 2 \\
&= 3^1(3^{k-1} - 1) + 2 \\
&= 3^k - 3 + 2 \\
&= 3^k - 1 \checkmark
\end{aligned}
$$

$S(k)$

This proves the statement $\square$

12/ "a is rational, b is irrational then $a+b$ is irrational"

<u>Proof</u> (by contradiction)

Assume $a+b$ is rational.

$a+b$ rational $\Rightarrow$ $a+b = c/d$

a is rational $\Rightarrow$ $a = e/f$

$$b = (a+b) - a$$
$$= c/d - e/f$$
$$= \frac{cf - ed}{df}$$

$\times$ since b is irrational.