CS 222 Lec 11

from last time

circuit fn = $(a + b \cdot c) \cdot (a' \cdot b)'$

$= (a + bc) \cdot (a + b')$

"absorption"

$a \cdot (a+b) = a$

$a + a \cdot b = a$

(dual)

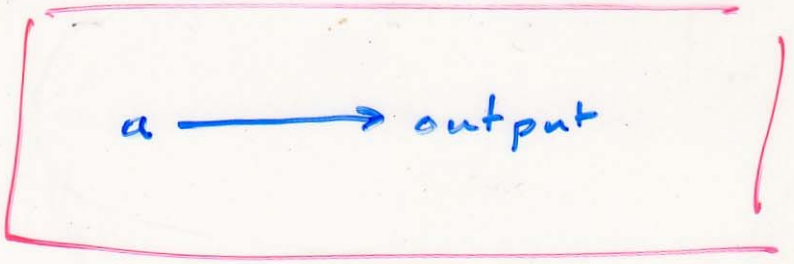
$= (a+bc) \cdot a + (a+bc) \cdot b'$

$= a + ab' + bcb'$

$= a + ab' + 0$

$= a + ab'$

$= a$



minimal circuit

(0 gates)

Boolean Functions

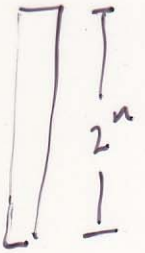
a	b	c	f
0	0	0	1
0	0	1	0
			⋮
1	1	1	⋮

there are $2^8 = 256$ possible boolean fns on 3 inputs.

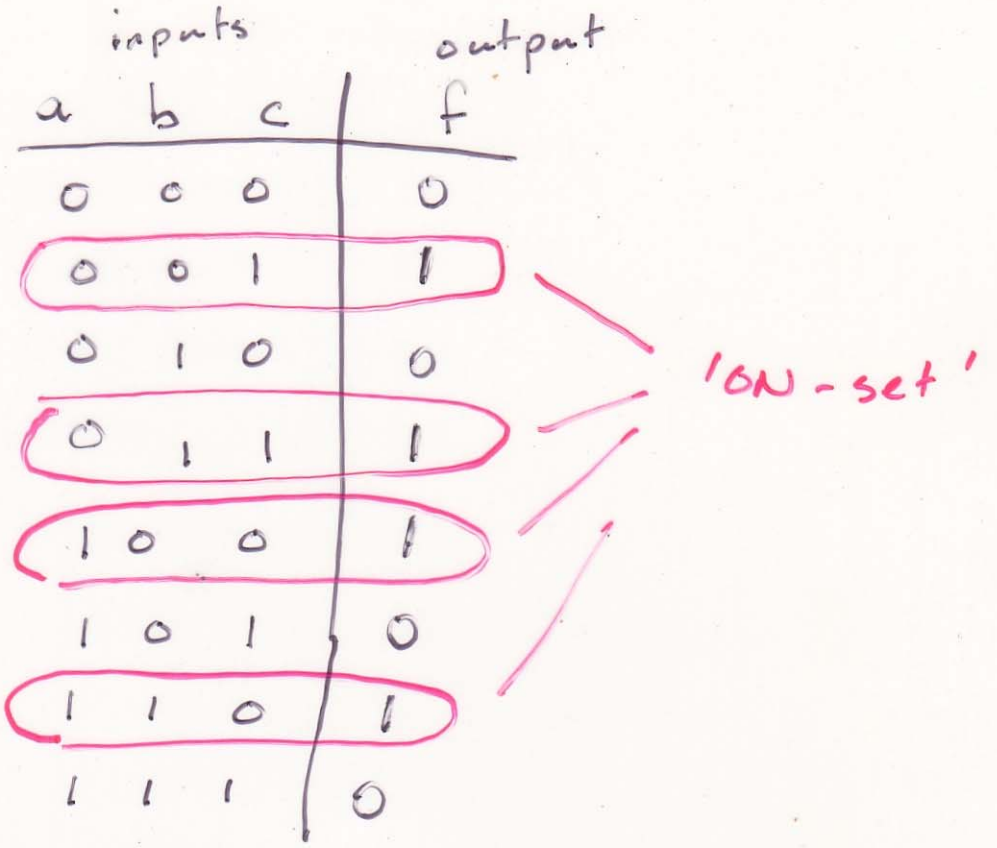
must specify this column

n inputs

$\Rightarrow$ must specify a binary column of length 2^n



$\Rightarrow$ there are 2^{2^n} such strings



Sum of Products Representation

$$f(a,b,c) = a'b'c + a'bc + ab'c' + abc'$$

⇓ Karnaugh Maps

Simplified S-O-P

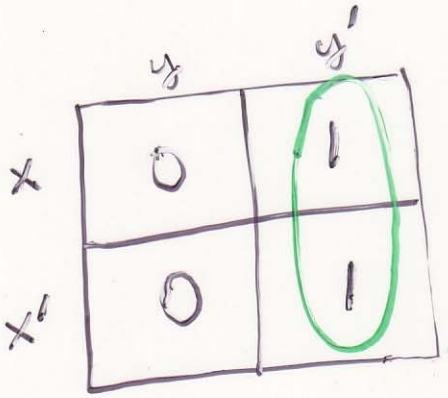
Karnaugh Maps

$f(x, y) = xy' + x'y'$

$= (x + x') \cdot y'$

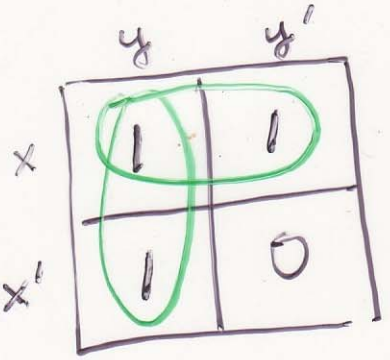
$= 1 \cdot y'$

$= y'$



simplified $f = y'$

$f(x, y)$



simplified $f = x + y$

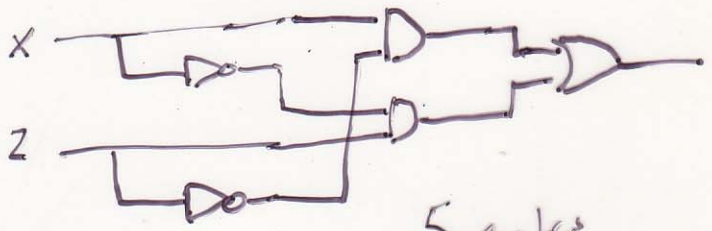
full S-o-P $= xy + x'y + xy'$

K-Maps for 3 variables

	yz	yz'	y'z'	y'z
x	0	1	1	0
x'	1	0	0	1

$f(x,y,z) = x'y'z + x'yz + xy'z' + xyz'$

simplified $f = xz' + x'z$



5 gates

with XOR-gate → 1 gate

f

	yz	yz′	y′z′	y′z
x	1	1	0	0
x′	1	1	1	1

$f = x' + y$

4 variable case {w, x, y, z}

	zw	zw′	z′w′	z′w
xy	0	0	0	0
xy′	1	1	1	1
x′y′	1	1	1	1
x′y	0	1	1	0