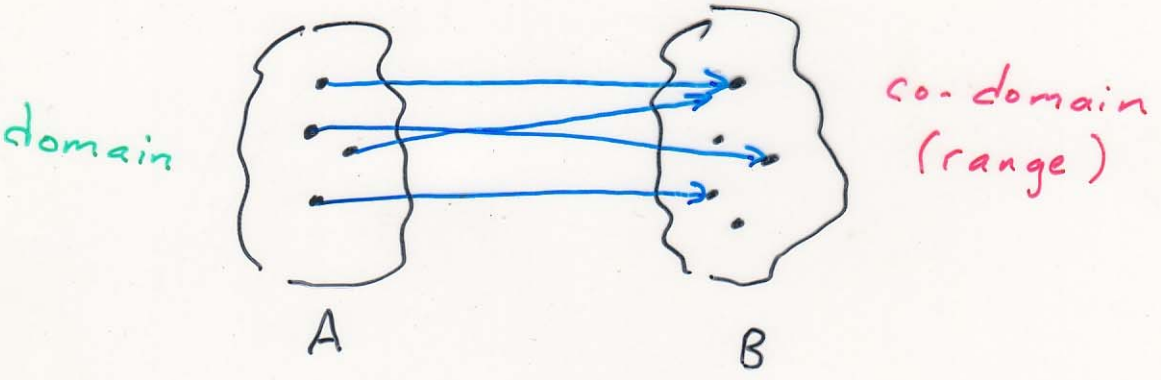
CS 222 Lec 12

Chapter 4: Functions + Relations

Functions

$$f: A \to B$$

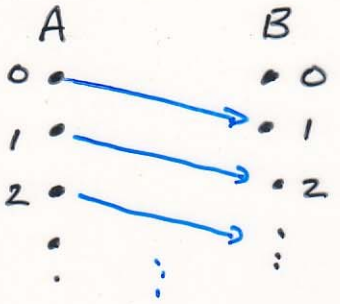
We say function f maps elements from A to elements in set B.



Key property: Each element in A is mapped to exactly one element in B.

e.g. $f: \mathbb{N} \to \mathbb{N}$

$f(x) = x + 1$



$f: \mathbb{Q} \to \mathbb{Q}$

$f(x) = \frac{1}{3}x + 5$

$f: \mathbb{Q}^{>0} \to \mathbb{R}$

$f(x) = \sqrt{x}$

from class: $\sqrt{2}$ is not rational

$f: A \to B$

spell-checker

A : set of all alphabetic strings

B : words in the dictionary

$f(s) \to w$ where w is the nearest dictionary word to s.

Pair-notation for functions

$f: A \to B$

$(x, f(x)) \in A \times B$

$f: \{ (x, f(x)) \mid x \in A \}$

is specified by

listing all pairs $(x, f(x))$

Binary Relations

A binary relation R

is a subset of pairs

of elements from $A \times B$

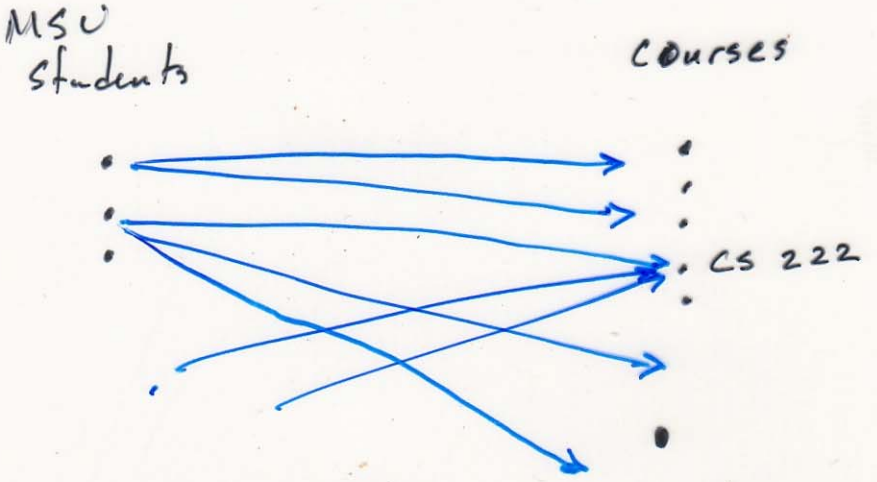
domain co-domain

Example

S = MSU students

C = all courses at MSU

$R_1 = \{(s, c) \mid$ student s is taking course $c\}$



R_1 is <u>not</u> a function!

Relations in General

- a subset of $A \times B \times C \times \ldots$

Inverse Relation

We define the inverse of a relation R as follows

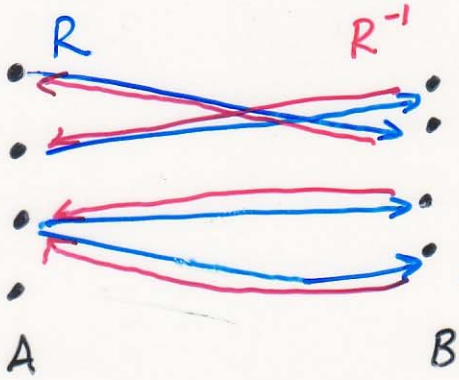
$$R^{-1} = \{ (y, x) \mid (x, y) \in R \}$$

If $R \subseteq A \times B$

Then $R^{-1} \subseteq B \times A$

$$R_1 = \{ (s, c) \mid \text{student } s \text{ was taking course } c \}$$

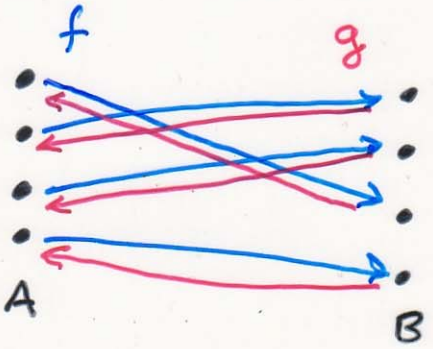
$$R_1^{-1} = \{ (c, s) \mid \text{course } c \text{ is being taken by student } s \}$$

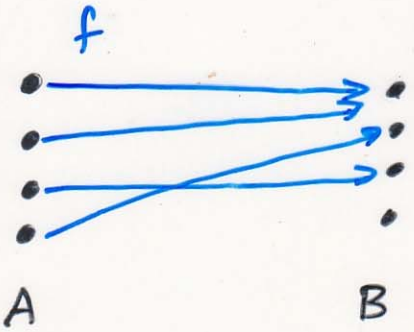


Inverses of Functions

A function f has an inverse g if

1) g is a function
2) $g(f(x)) = x$





f is not invertible

$f: \mathbb{Z} \to \mathbb{Z}$

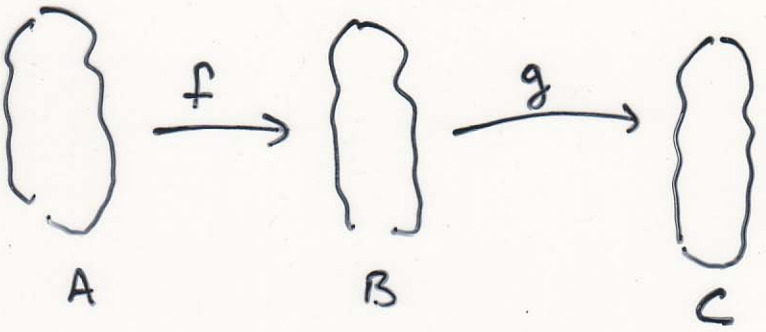
$f(x) = x + 1$

$g(x) = x - 1$

$g(f(x)) = g(x+1) = (x+1) - 1 = x$

g is the inverse of f

Composition of Functions



We can 'compose' these functions

$$g \circ f : A \to C$$

$$g \circ f(x) = g(f(x))$$

Example f is invertible.

$$f^{-1} \circ f(x) = x$$

$f^{-1} \circ f$ is the identity function