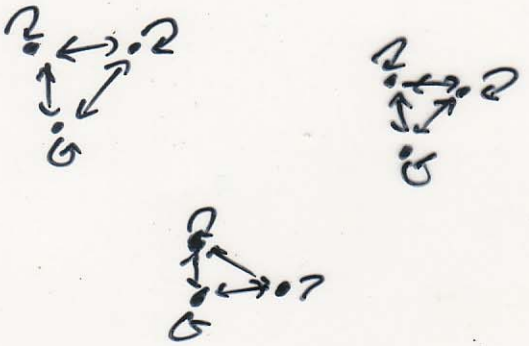
CS 222 Lec 15

Equivalence Relations

R is an equ. relation iff

$\exists$ partition $P = \{ P_1, P_2, \ldots \}$

$(x, y) \in R \iff x, y \in P_i$

A 

$\mathbb{Z}$ = integers

$(x, y) \in R \iff x - y$ is even.

$P = \{$ even numbers, odd numbers $\}$

$(x,y) \in R \iff k \mid x-y$

Thm R is an eqv. relation

$\iff$ 1) R is reflexive

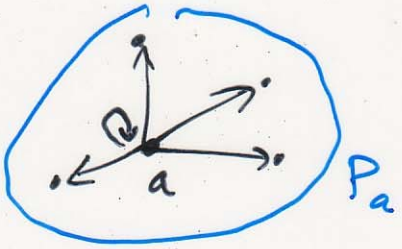
2) R is symmetric

3) R is transitive.

Pf

"$\Leftarrow$" Suppose R is reflexive, symmetric + transitive.

$\forall a \in A$, $P_a = \{x \in A \mid (a,x) \in R\}$



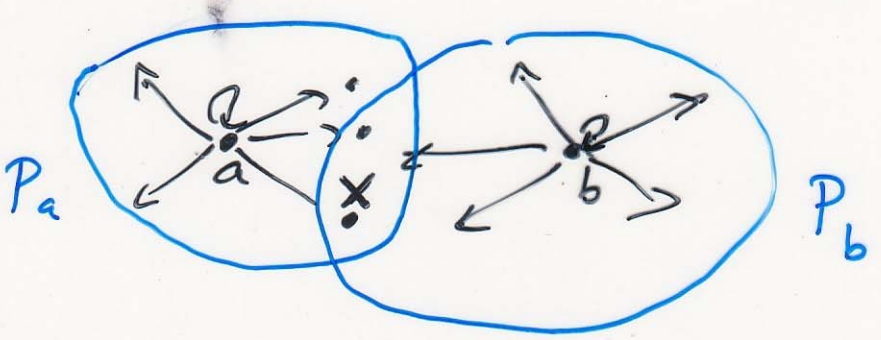
Let $P = \{P_a\}$ for all $a \in A$.

Claim: P is a partition

1) Each $P_a \in P$ is nonempty.

yes, since $a \in P_a$

2) $P_a \cap P_b \neq \emptyset \Rightarrow P_a = P_b$



we know

$u \in P_a \Rightarrow (u, x) \in R$

$v \in P_b \Rightarrow (x, v) \in R$

by transitivity $\Rightarrow (u, v) \in R$
$(v, u) \in R$

$\therefore P_a = P_b$

3) $\bigcup_{a \in A} P_a = A$ ✓

$(x, y) \in R \iff x$ and y belong to the same P_a.

Combinatorics (Chp. 5)

"Counting problems"

How many ways to do something?

Questions

are repeats allowed?

does order matter?

Order matters?

repeats allowed?		yes	no
	yes	ordered list	bag (multiset)
	no	permutation	set

Rules for Counting

Rule of Products

Task A first ⇒ Task B second

(assume that choice of A doesn't affect B)

Task A → m ways

Task B → n ways

# of ways to do A followed by B $= m \cdot n$

A/323

$26 \cdot 1000 = 26,000$