# CS 222 Lec 16

"Combinatorics"

- counting problems

## Rules of Products

A 123 license plates.

$26 \cdot 10^3 = 26{,}000$

key point:
choice of letter
is independent
of number

$$|A \times B| = |A| \cdot |B|$$

# Rule of Sums

# of ways to a task
(assume can done
by either A or B)

$= |A| + |B|$

example

order lunch:

1) sandwich 5

2) soup 6

$5 + 6 = 11$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

## Counting Ordered Lists.

$\{1, \ldots n\}$ objects

# of ordered lists of these objects of length $r$

$$= n^r$$

$$\underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{r} = n^r$$

## Counting Permutations

$\{1, \ldots, n\}$ objects

# of ways to form a permutation of length $r$

$$= n \cdot (n-1) \cdot (n-2) \cdots (n-r+1)$$

$= P(n,r)$

= # of permutations of $n$ objects of length $r$.

Special case ($r = n$)

$P(n,n) = n(n-1)\cdot(n-2)\cdots 2\cdot 1$

$= n!$

$$P(n,r) = \frac{n!}{(n-r)!}$$

## Combinations

- like a permutation, except order doesn't matter

e.g. (dealing a hand in card game)

$\{1, 2, \ldots, n\}$ objects

How many ways to choose a set of $r$ objects?

$$C(n,r) = \frac{n!}{(n-r)!\, r!}$$

$$= P(n,r)/r!$$

# of 5-card hands

$$= C(52,5)$$