CS 222 Lec 18

n objects, picking r

Summary

1) permutation (order important, no repeats) $\rightarrow P(n,r) = \frac{n!}{(n-r)!}$

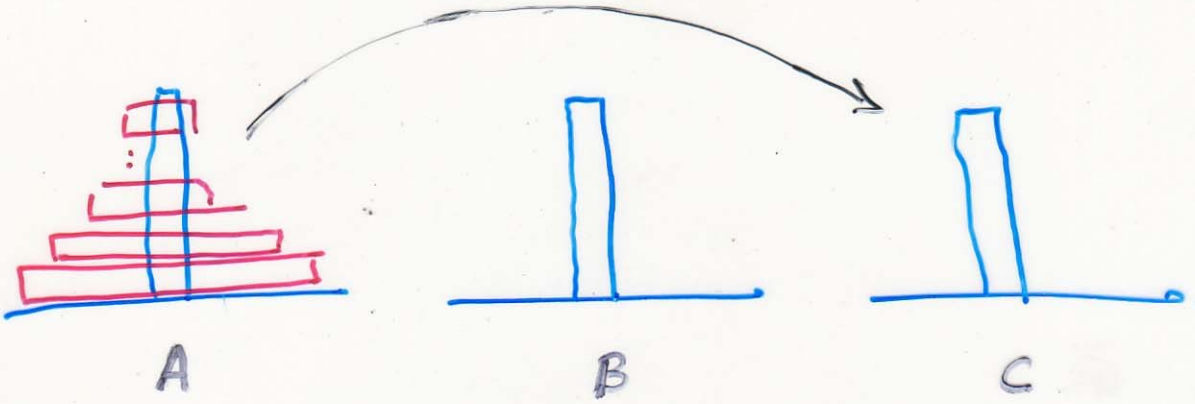
2) ordered list (order important, repeats allowed) $\rightarrow n^r$

3) combinations (order not important, soln = set, repeats not allowed) $\rightarrow \binom{n}{r}$

4) bags / multisets (set, do allow repeats) $\rightarrow \binom{n+r-1}{r}$

Recursive Counting

Towers of Hanoi



recursive soln (A → C)

- move n-1 disks to B H_{n-1} moves
- move last disk from A to C 1 move
- move n-1 disks from B to C H_{n-1} moves

How many moves required?

H_n = # moves needed for n disks.

$$H_n = 2 \cdot H_{n-1} + 1$$

$$H_1 = 1$$

$H_1 = 1$

$H_2 = 3$

$H_3 = 7$

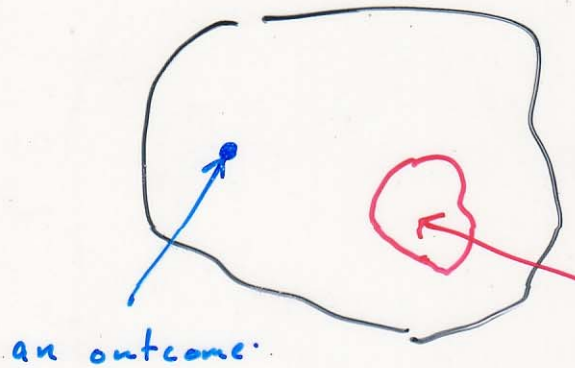
$H_4 = 15$

$\vdots$

$$H_n = 2^n - 1 \quad *$$

Probability

sample space: set of possible outcomes (finite / infinite)



e.g. flipping a coin

sample space = $\{H, T\}$

"head" event H = $\{H\}$

e.g. rolling two dice

sample space = $\{(1,1), (1,2) \ldots (6,6)\}$

event = $\{(x,y) \mid x+y=7\}$

uniform distribution:

$$Pr(E) = \frac{|E|}{|S|}$$

E = event

S = sample space.