CS 222 Lec 19

- sample space : set of outcomes
- event: subset of the sample space.

$$\Pr(E) = \frac{|E|}{|S|}$$ (for uniform distributions)

↓

every outcome in S is equally likely

We will just be concerned with <u>finite</u> <u>sample</u> spaces.

Observe:

$$\Pr(S) = 1$$

$$\forall E, \quad 0 \le \Pr(E) \le 1$$

$$\Pr(\overline{E}) = \Pr(S \setminus E)$$

$$= \frac{|S \setminus E|}{|S|}$$

$$= \frac{|S| - |E|}{|S|}$$

$$= 1 - \frac{|E|}{|S|} = 1 - \Pr(E)$$

$$\Pr(E) + \Pr(\overline{E}) = 1$$

Birthday Problem

$S = \{ 30 \text{ random birthdays} \}$ ← assume no Feb 29 birthdays.

E = there are two people that have same birthday

$$\Pr(E) = \frac{|E|}{|S|}$$

$$\Pr(E) = 1 - \Pr(\bar{E})$$

$$= 1 - \frac{|\bar{E}|}{|S|}$$

$$|S| = 365^{30}$$

$$|\bar{E}| = \underbrace{365 \cdot 364 \cdot 363 \cdots \cdot 336}_{30 \text{ factors}}$$

$$\Pr(E) = 1 - \frac{365 \cdot 364 \cdots 336}{365^{30}}$$

$\approx .72$ (by experiment)

Die Game Problem

John's die : 4-sided $\overset{A=}{\{1,2,3,4\}}$

Jessie's die : 6-sided $\underset{B=}{\{1,2,\dots 6\}}$

$$S = A \times B$$

E = John wins.

$$= \{(x,y) \in S \mid x > y\}$$

(2,1)
(3,1)
(3,2)
(4,1)
(4,2)
(4,3)

$$Pr(E) = \frac{|E|}{|S|} = \frac{6}{24}$$

$$= \underline{.25}$$

Solitaire Game (pg 445)

game rules

1. shuffle deck
2. pull out adjacent pairs that have same color
3. win: if no cards are left.

   lose: otherwise.

$Pr(\text{win}) = ?$

26 $\bullet$'s

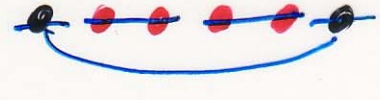
26 $\bullet$'s

$S = \{ \underline{\bullet}\,\underline{\bullet}\,\underline{\bullet}\,\underline{\bullet} \ \ldots \ \underline{\bullet}\,\underline{\bullet}\,\underline{\bullet} \}$

$E =$

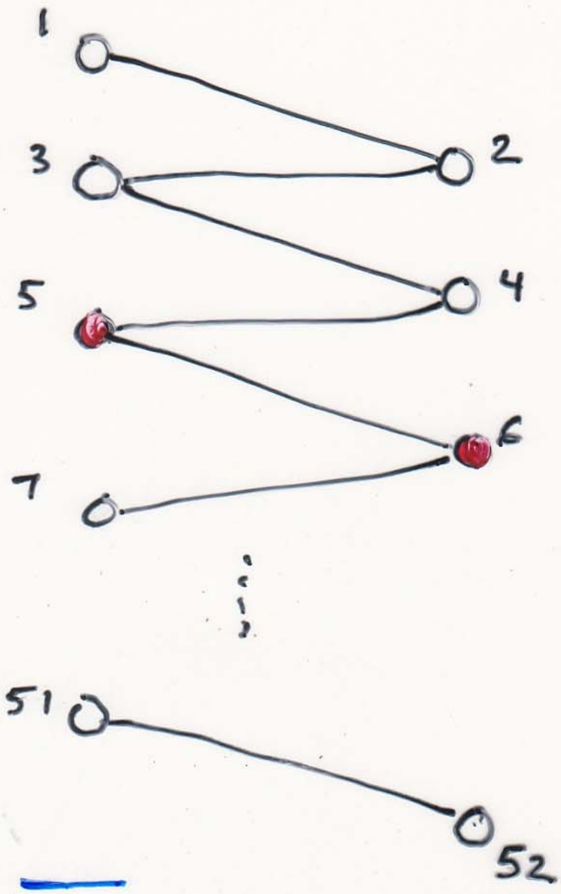
$$|S| = \binom{52}{26}$$

$|E| = ?$



$\to$ win?

$\binom{26}{13}$ ways to arrange left hand side



$\binom{26}{13}$ ways.

Observe # of red cards on this side $\overset{\text{}}{=}$ # of red cards on this side

$\|$ 13 $\qquad\qquad$ $\|$ 13

E = set of winning decks

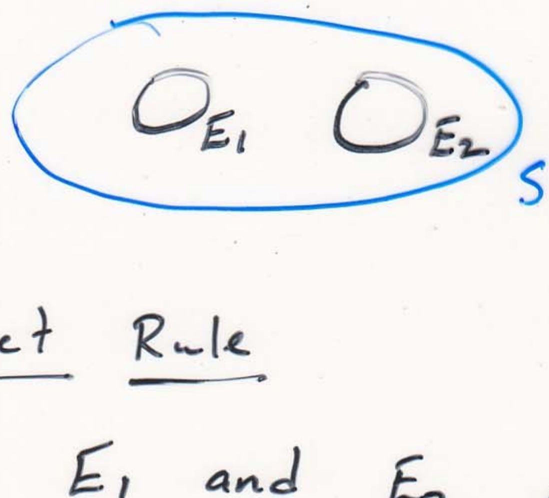
$$|E| = \binom{26}{13} \cdot \binom{26}{13}$$

$$Pr(\text{win}) = \frac{\binom{26}{13}^2}{\binom{52}{26}} = .22$$

Sum Rule for Probability

E_1, E_2 that are disjoint

$$Pr(E_1 \text{ or } E_2) = Pr(E_1) + Pr(E_2)$$



Product Rule

if E_1 and E_2 are independent

$$Pr(E_1 \text{ and } E_2) = Pr(E_1) \cdot Pr(E_2)$$

E_1 = head on 1st flip
E_2 = tail on 2nd flip.