CS 222 Lec 2

Propositional Logic

Defn A proposition is a statement that is either true or false.

p "This is CS 222" T

q "My age is 19" F

A propositional variable stands for a prop.

p = true,

q = false.

Logical Operators

AND, OR, NOT

Symbol form: $\wedge$ $\vee$ $\neg$

Ex $(p \wedge q)$

"Compound" proposition

($p \vee q$, $\neg p \wedge q$, ...

allow us to build more complex propositions from simple ones.

Truth Tables

Ex. $p \vee \neg q$

p	q	$\neg q$	$p \vee \neg q$
F	F	T	T
F	T	F	F
T	F	T	T
T	T	F	T

$\oplus$ = exclusive-OR

p	q	$p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$
F	F	F
F	T	T
T	F	T
T	T	F

<u>Defn</u> Two logical propositions are <u>equivalent</u> if and only if the have the same truth values in each row of a truth table. ($\equiv$)

<u>Ex</u> $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
F	F	T	T
F	T	F	F
T	F	F	F
T	T	F	F

equal

$$\boxed{\begin{aligned} \neg(p \vee q) &\equiv \neg p \wedge \neg q \\ \neg(p \wedge q) &\equiv \neg p \vee \neg q \end{aligned}}$$

De Morgan's laws *

"distributive property"

<u>Ex</u>

1. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
2. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	T	T
T	F	F	T	T
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

equal, so $\equiv$

<u>Ex</u> Another knight / knave problem...

<u>John</u> says "If Bill is a knave then I am a knight"

[we can write this logically as
$\neg b \to j \quad (\equiv b \vee j)$]
↑ will discuss more

<u>Bill</u> says "We are different"

[$(j \wedge \neg b) \vee (\neg j \wedge b)$]

<u>Observation!</u>

Either John is a knight and made a true statement
OR John is a knave and lied

so,

$$(j \wedge (b \vee j)) \vee (\neg j \wedge \neg (b \vee j))$$

(corrected version)

$$(j \wedge (b \vee j)) \vee (\neg j \wedge \neg (b \vee j))$$

$$\equiv (j \wedge (b \vee j)) \vee (\neg j \wedge (\neg b \wedge \neg j))$$

$$\equiv (j \wedge (b \vee j)) \vee (\neg b \wedge \neg j)$$

$$\equiv (j \wedge (b \vee j)) \vee \neg (b \vee j)$$

$$\equiv (j \vee \neg (b \vee j)) \wedge ((b \vee j) \vee \neg (b \vee j))$$

$$\equiv (j \vee \neg (b \vee j)) \wedge T$$

$$\equiv (j \vee \neg (b \vee j))$$

$$\equiv (j \vee (\neg b \wedge \neg j))$$

$$\equiv (j \vee \neg b) \wedge \underbrace{(j \vee \neg j)}_{T}$$

$$\equiv \boxed{j \vee \neg b}$$

this must be true regardless of whether John is telling the truth or lying.

Observation 2

Bill is either a knight and his statement is true or a knave and his statement is false.

So,

$$(b \wedge [(j \wedge \neg b) \vee (\neg j \wedge b)])$$
$$\vee\ (\neg b \wedge \neg [(j \wedge \neg b) \vee (\neg j \wedge b)])$$

$$\equiv ((\underbrace{b \wedge (j \wedge \neg b)}_{F})) \vee (b \wedge (\neg j \wedge b))$$
$$\vee\ (\neg b \wedge \neg (j \wedge \neg b) \wedge \neg(\neg j \wedge b))$$

$$\equiv (b \wedge \neg j) \vee (\neg b \wedge (\neg j \vee b) \wedge \neg(b \wedge \neg j))$$

$$\equiv (b \wedge \neg j) \vee (((\neg b \wedge \neg j) \vee \underbrace{(\neg b \wedge b)}_{F}) \wedge \neg(b \wedge \neg j))$$

$$\equiv (b \wedge \neg j) \vee ((\neg b \wedge \neg j) \wedge \neg(b \wedge \neg j))$$

$$\equiv ((b \wedge \neg j) \vee (\neg b \wedge \neg j)) \wedge (\underbrace{(b \wedge \neg j) \vee \neg(b \wedge \neg j)}_{T})$$

$$\equiv (b \wedge \neg j) \vee (\neg b \wedge \neg j)$$

$$\equiv \underbrace{(b \vee \neg b)}_{T} \wedge \neg j$$

$$\equiv \boxed{\neg j}$$ is true

We can now combine Obs. 1 and Obs. 2:

$$(j \vee \neg b) \wedge \neg j$$

$$\equiv \underbrace{(j \wedge \neg j)}_{F} \vee (\neg b \wedge \neg j)$$

$$\equiv \boxed{\neg b \wedge \neg j}$$

This is the answer: Both John and Bill are <u>knaves</u>.

[OK: Truth tables are easier! :)]