CS 222 Lec 20

Probability cont.

Sum Rule (general version)

$Pr(E_1 \text{ or } E_2)$

$= Pr(E_1) + Pr(E_2) - Pr(E_1 \text{ and } E_2)$

e.g. Given a standard shuffled deck of cards, what is the chance of the top 3 cards being either 3 face cards <u>or</u> 3 cards from the same suit?

Let E_1 : 3 face aards on top

E_2 : top 3 card from same suit

$$Pr(E_1) = \frac{\binom{16}{3}}{\binom{52}{3}}$$

$$Pr(E_2) = \frac{\binom{4}{1}\cdot\binom{13}{3}}{\binom{52}{3}}$$

choose suit

choose 3 face cards

$$Pr(E_1 \cap E_2) = \frac{\binom{4}{1}\cdot\binom{4}{3}}{\binom{52}{3}}$$

$$Pr(E_1 \text{ or } E_2) = \frac{\binom{16}{3} + \binom{4}{1}\cdot\binom{13}{3} - \binom{4}{1}\binom{4}{3}}{\binom{52}{3}}$$

$$\simeq .0615$$

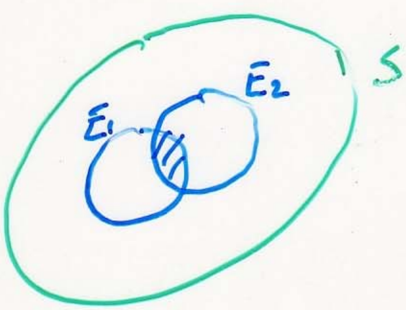
Product Rule

E_1, E_2 : independent

$$\Rightarrow \Pr(E_1 \wedge E_2) = \Pr(E_1) \cdot \Pr(E_2)$$

E_1, E_2 : could be dependent:

$$\Pr(E_1 \wedge E_2) = \Pr(E_1) \cdot \Pr(E_2 \mid E_1)$$



the conditional prob. of E_2 occurring given that E_1 occurs.

$$= \Pr(E_2) \cdot \Pr(E_1 \mid E_2)$$

$$\Pr(E_1 \mid E_2) = \frac{\Pr(E_1 \wedge E_2)}{\Pr(E_2)}$$

E_1 = 1^{st} marble is red

E_2 = 2^{nd} marble is red

(original bag contains: 3 red, 5 white, 8 green)

$$Pr(E_1 \wedge E_2) = Pr(E_1) \cdot Pr(E_2 / E_1)$$

$$= 3/16 \cdot 2/15 = 6/240$$

$$= \cancel{}$$

$$= 1/40$$

<u>Bernoulli</u> <u>Trials</u>

- flip a biased coin n times, what is # of heads?

- let p = probability of getting a 'head'

$$\underbrace{\underline{T}\ \underline{H}\ \underline{T}\ \underline{H}\ \underline{H}\ \underline{\ldots}\ \underline{\ \ }\ \underline{\ \ }\ \underline{\ \ }}_{n \text{ flips}}$$

$$\Pr(\text{exactly } k \text{ heads}) = \binom{n}{k} p^k (1-p)^{n-k}$$

Bernoulli distribution

Given we roll a fair die 10 times, what's the chance we get exactly five 6's?

$$\binom{10}{5}\left(\frac{1}{6}\right)^5\left(\frac{5}{6}\right)^5 \approx 0.013$$

Expected Value of a Random Variable

Random Variable

S = numerical values

X = # of games needed to win series

What is average value of x?

$$E(x) = \sum_{x \in S} x \cdot Pr(x)$$

expected value of x

e.g. X = value of die roll

$$E(x) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \cdots + 6 \frac{1}{6}$$

$$= \frac{1}{6}(1+2+3+4+5+6) = \boxed{3.5}$$

<u>Game</u> cost \$2 to play:

- roll 2 dice
- win \$5 for every '6' that appears

let X = # of 6's that occur

$$x \in \{0, 1, 2\}$$

$$E(x) = 0 \cdot Pr(0) + 1 \cdot Pr(1) + 2 \cdot Pr(2)$$

$$= 1 \cdot \left(\frac{10}{36}\right) + 2\left(\frac{1}{6}\right)^2$$

$$= \frac{12}{36}$$

$$E(pay) = \$5 \cdot \frac{12}{36} = \$\frac{60}{36} = \$1.67$$

Baseball Series (best of 5 series)

teams R + G

$Pr(R \text{ wins}) = p$

$X = \#$ length of series

$E(X) = ?$

$X = \{3, 4, 5\}$

Pr (X = 3) possible outcomes

R R R p^3

G G G $(1-p)^3$

Pr (X = 4) possible outcomes

_ _ _ R $\binom{3}{2} p^3 (1-p)$

_ _ _ G $\binom{3}{2} (1-p)^3 p$

Pr (X = 5)

_ _ _ _ R $\binom{4}{2} p^3 (1-p)^2$

_ _ _ _ G $\binom{4}{2} (1-p)^3 p^2$

$E(X) = 3 \cdot Pr(3) + 4 \cdot Pr(4) + 5 \cdot Pr(5)$