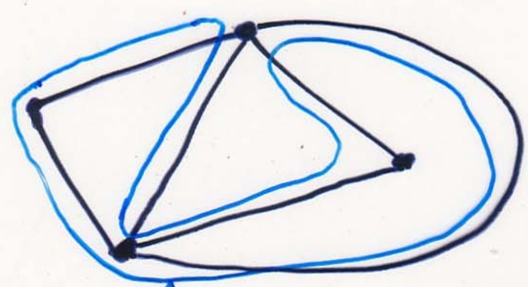
CS 222 Lec 22

Graphs

$$G = (V, E)$$

vertices edges

Eulerian Graphs



Eulerian circuit

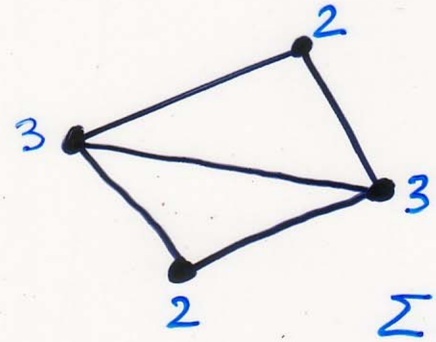
Thm: A graph is Eulerian iff all vertices have even degree.

deg(v)=4

Properties of Graphs

Thm If $G = (V, E)$ is undirected

Then
$$\sum_{v \in V} \deg(v) = 2 \cdot |E|$$



$$\sum \deg(v) = 10$$
$$= 2 \cdot |E|$$

Proof

Every edge contributes $+1$ to the degrees of two vertices, so increases $\sum \deg(v)$ by $+2$.

Corollary

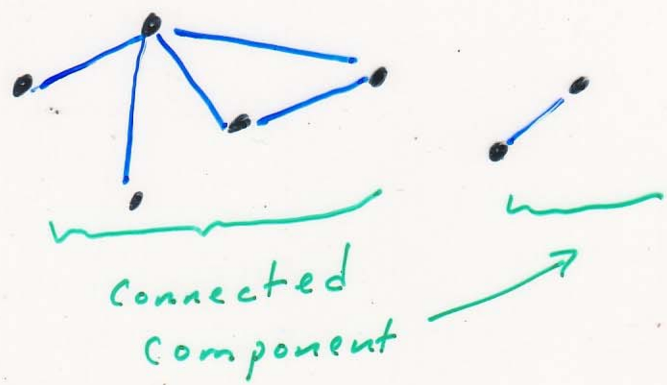
In any graph, the number of odd degree vertices must be even.

Question is it possible to have a graph with the following degree sequence:

$$2, \underline{3}, \underline{3}, 4, \underline{5} \,?$$

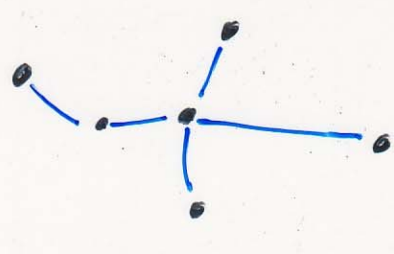
odd $\Rightarrow$ so no.

Defn A graph is <u>connected</u> if $\forall a, b \in V$, there is a path in G from a to b



Defn A graph is a tree if it is:

1) connected
2) does not contain any cycles.

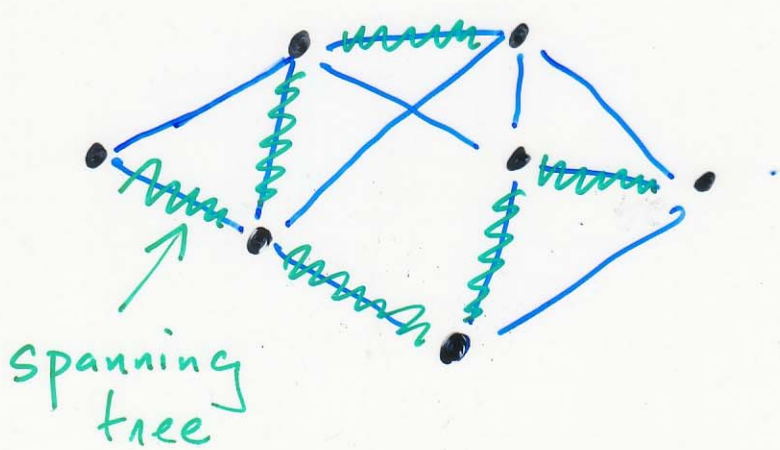


Observe:

Any tree T with n vertices has $n-1$ edges.

Spanning Trees

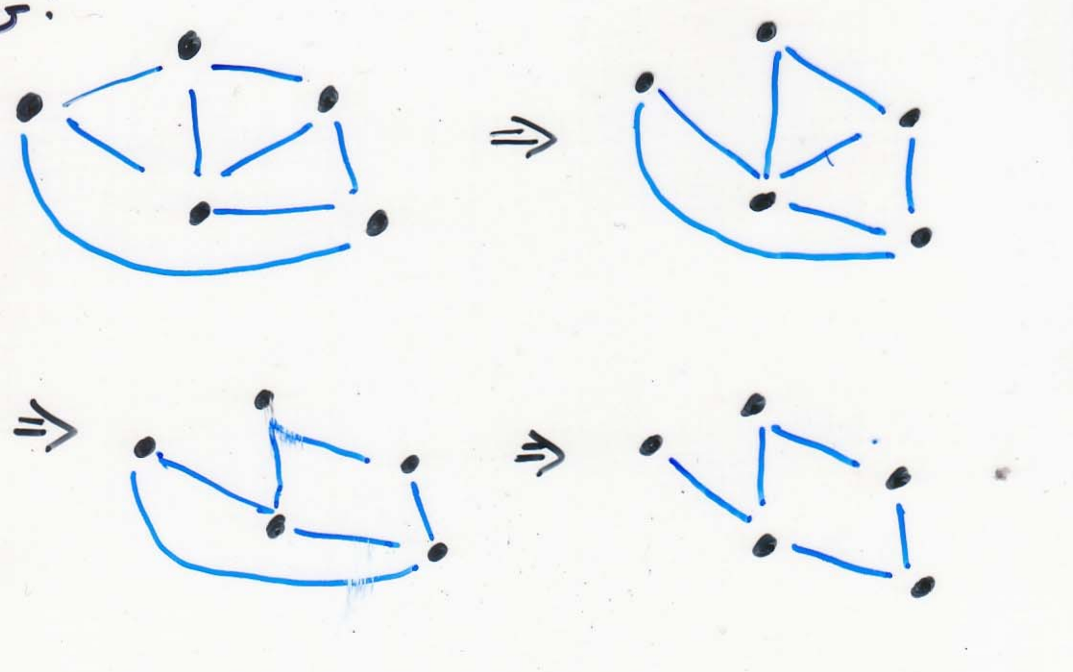
A spanning tree T for a graph G is a subset of the edges of G that forms a tree, and each vertex in G is connected by T.



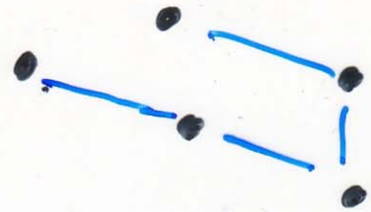
Algorithm to find Spanning Trees.

1) Let G be input graph (connected)
2) Find a cycle in G, delete any edge in cycle.
3) Repeat step 2. until no cycles remain.

G:

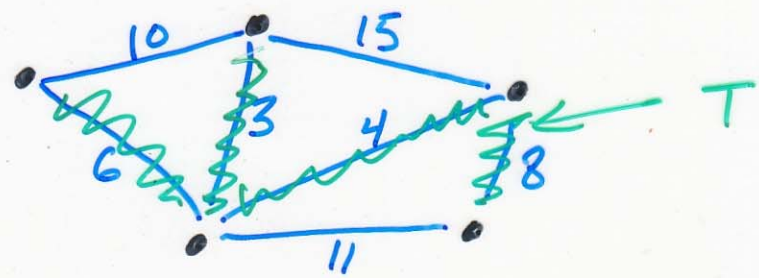


$\Rightarrow$



Minimum Spanning Trees

given: weighted undirected graph



$\text{cost}(T) = 22.$

$$\text{cost}(T) = \sum_{e \in T} w(e)$$

$w(e)$ = weight of edge e.

Problem: find a MST T^* for G

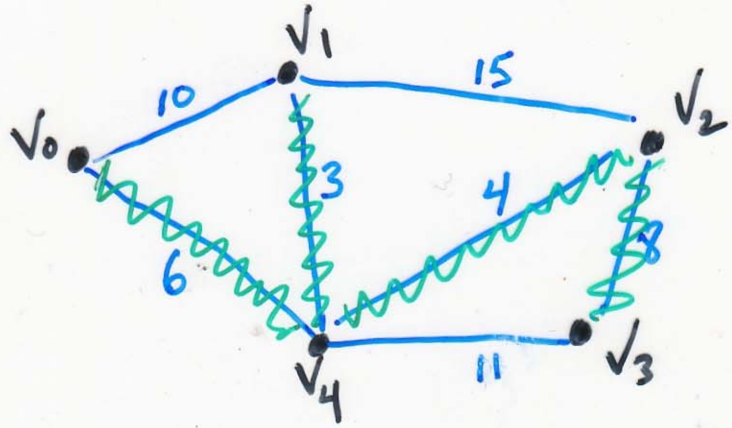
where

$$\text{cost}(T^*) \le \text{cost}(T)$$

T is any spanning tree.

Prim's algorithm

"grow a MST"



1) start by picking a random vertex.

$T = \{v_0\}$

2) repeat: find the cheapest edge that goes outside of T and add to T

3) repeat until all vertices belong to T.

Another MST alg:

Kruskal's algorithm

1. sort all edges be weight.

$$w(e_1) \le w(e_2) \le w(e_3) \le \cdots \le w(e_m)$$

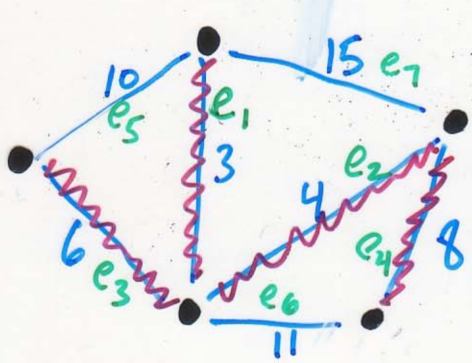
$T = \{\}$

2. for $i = 1$ to m

   add e_i to T

   provided doesn't create a cycle in T.

3. return T.



Question:

Suppose G has all unique edge weights? Is G's MST unique?

yes.