CS 222 Lec 23

Graph Isomorphism

We say two graphs G_1 and G_2 are isomorphic if the have the same structure.

$G_1 = (V_1, E_1)$
$G_2 = (V_2, E_2)$

$a \longleftrightarrow 1$
$c \longleftrightarrow 3$
$b \longleftrightarrow 4$
$d \longleftrightarrow 2$

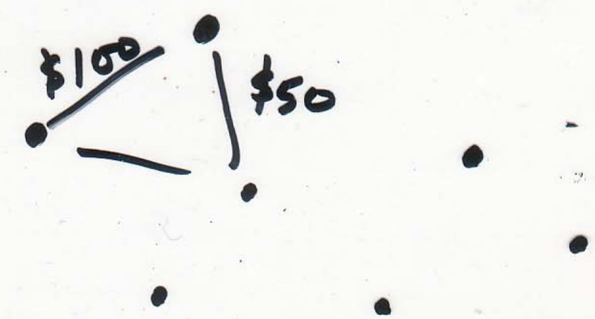
G_1 + G_2 are isomorphic.

$\Longleftrightarrow$ $\exists$ a 1-1, onto fn $f: V_1 \to V_2$
s.t. $(u,v) \in E_1 \Longleftrightarrow (f(u), f(v)) \in E_2$

Interesting fact:

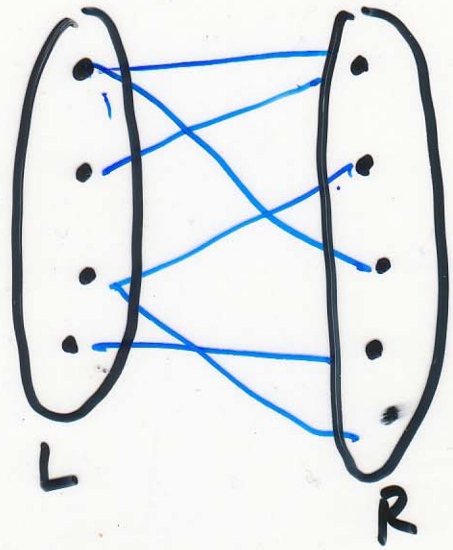
- "NP-complete" problem
- best known algorithms are brute-force, and require exponential time.

TSP



- salesman must every city exactly once.
- find cheapest 'tour' for the salesman.
- NP-complete.

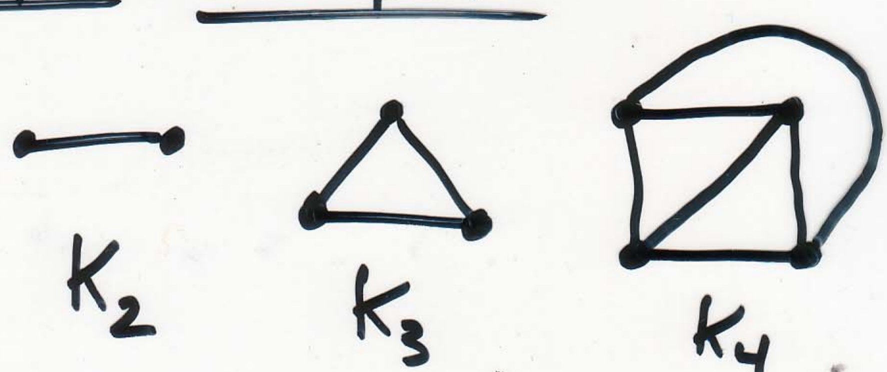
Bipartite Graphs



$G = (V, E)$ is bipartite

iff $E \subseteq L \times R$

Complete Graphs

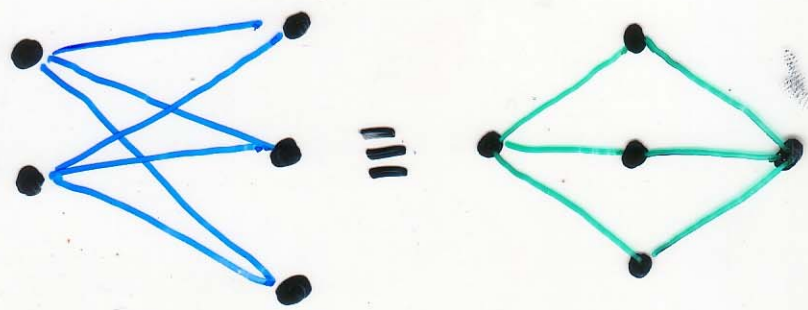


K_n = complete graph on n vertices.

Complete Bipartite Graphs

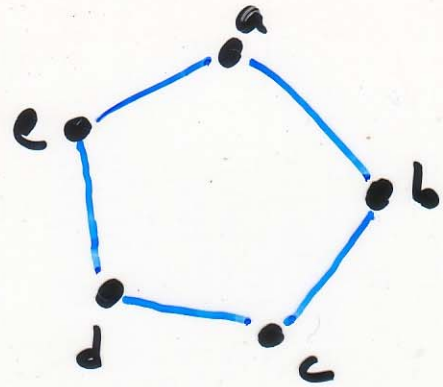
$K_{n,m} = |L| = n \quad |R| = m$

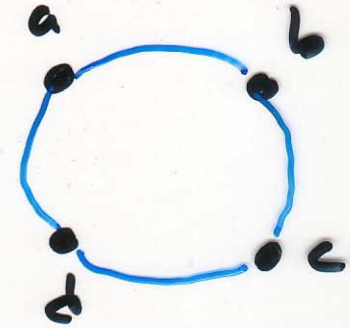
$K_{2,3}$:

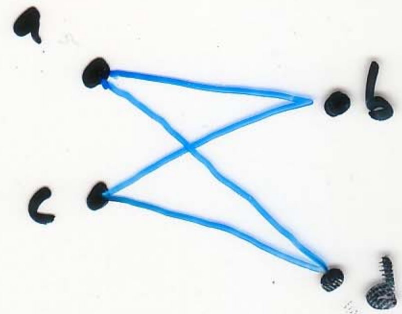


Planar Graph

any graph that can be drawn so that no edges cross.







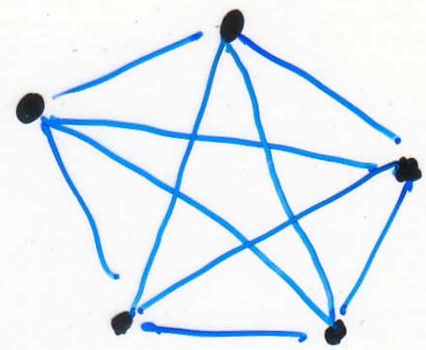
C_n = circle graph with n vertice

C_n is bipartite $\iff$ n is even.

<u>Kuratowski's</u> <u>Thm</u> (~1930)

A graph is planar $\iff$ it doesn't contain a copy of K_5 or $K_{3,3}$

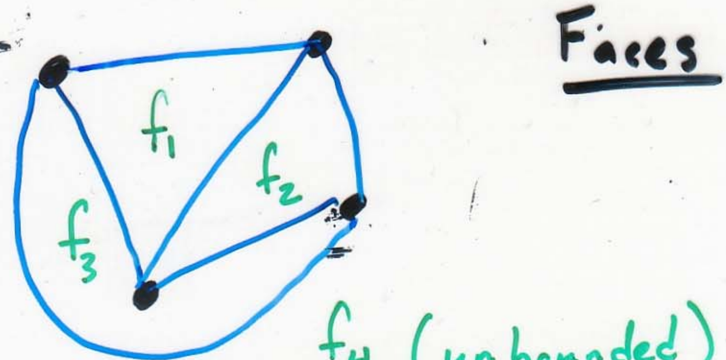
K_5



K_5 is **not** planar.

Euler's Formula

if we have a planar, connected graph G.

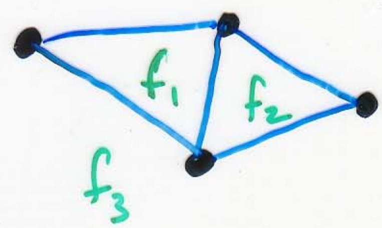


Faces

$$\boxed{V + F = E + 2}$$

$$4 + 4 = 6 + 2$$

Let G be a planar graph, every edge in G is part of cycle.



face-size(f) = # of edges surrounding f

Lemma: $\sum$ face-sizes $= 2 \cdot E$

Lemma: suppose size of smallest cycle is $= k$

$\Rightarrow \sum$ face-sizes $\geq k \cdot F$

$\Rightarrow E \geq \frac{k}{2} \cdot F$

Proof that $K_{3,3}$ is not planar

by contradiction:

Suppose $K_{3,3}$ is planar

- $\underset{6}{V} + F = \underset{9}{E} + 2 \Rightarrow F = 5$
- min cycle length $= k = 4$
- by prev. lemma. $E \geq \frac{k}{2} \cdot F$

$$= 2 \cdot 5$$

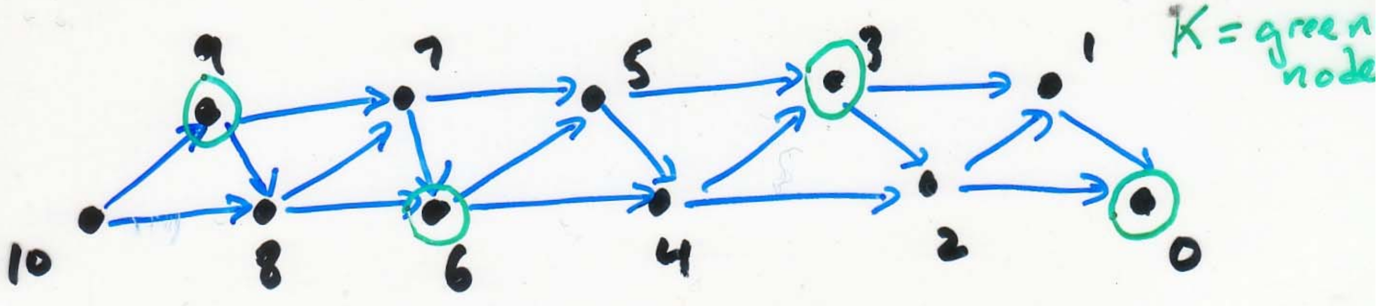
$$= 10$$

since $E = 9$

Graphs in Games

- use graphs to represent states + legal moves in a game.

Ex.
- 2-person game
- pile of 10 stones
- each player can remove 1-2 stones
- player to remove last stone wins.



Defn the kernel of a 2 player game graph.

K = set of nodes

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1. winning node(s) $\in K$
2. $\forall$ node $n \notin K$, $\exists$ an edge from n into a node in K.
3. there are no edges between nodes in K.

K = good nodes

Nim

- 2 players (alternate turns)

piles of stones

turn

- select a pile
- remove any # of stones from that pile.

win

- if you remove last stone present.

Read textbook for a description of the **kernel**.