CS 222 - Lec 3

Implications + Predicates

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another logical operator

$p \to q$

p	q	$p \to q$	
F	F	T	"vacuously true"
F	T	T	
T	F	F	
T	T	T	

$p \to q$

$$p \to q \equiv \neg p \vee q$$

$$\equiv \boxed{\neg q \to \neg p}$$

"contrapositive"

p	q	$\neg p$	$\neg q$	$\neg q \to \neg p$
F	F	T	T	T
F	T	T	F	T
T	F	F	T	F
T	T	F	F	T

Predicates

A predicate is a true / false statement involving one (or more) variables.

$P(x)$ = x is enrolled in CS 222.

domain = people in the classroom

$O(n)$ = n is an odd integer

domain = integers ($\mathbb{Z}$)

$O(1)$ is true

$O(2)$ is false

⋮

$E(n)$ = n is even

Quantifiers

"for every" (universal quantifier)

$$\forall n \in \mathbb{Z} \quad O(n) \vee E(n)$$

"there exists" (existencial quantifier)

$$\exists n \in \mathbb{Z} \quad E(n)$$

$P(x)$ = x is enrolled in CS 222

$R(x)$ = x is a sophomore

domain = people in this class

$$\exists x, P(x) \wedge R(x)$$

Negations of Quantifiers

$$\neg[\exists x \; P(x) \wedge R(x)]$$

$$\equiv \forall x \neg(P(x) \wedge R(x))$$

$$\equiv \forall x (\neg P(x) \vee \neg R(x))$$

domain = even numbers

$\neg \exists x \; O(x)$ is true

$\equiv \forall x \neg O(x)$

Rules of Negation

$\neg \exists s \; P(s) \equiv \forall s \neg P(s)$

$\neg \forall s \; R(s) \equiv \exists s \neg R(s)$

Ex domain = $\mathbb{Z}$

$\forall x \forall y \quad x+y=10$ false

$\forall x \exists y \quad x+y=10$ true

($y = 10-x$)

$\exists x \forall y \quad x+y=10$ false

$\exists x \exists y \quad x+y=10$ true

Remark $\exists!$ = "there exist uniquely"

$\forall x \exists! y, x+y=10$

$\exists! x \forall y \quad x \cdot y = 0$

is true.