CS 222 Lec 7

<u>Claim</u> In any single-elimination basketball tournament with n teams exactly $n-1$ games must be played to determine a winner.

<u>Pf</u> by induction on n.

<u>Step 1</u> (basis) $n=1$, 0 games needed ✓

<u>Step 2</u> (inductive step): Show that

$$\text{statement}_{n=m-1} \Rightarrow \text{statement}_{n=m}$$

let there be m teams at the start, after 1 game, $m-1$ teams remain. By inductive hypothesis, we another $m-2$ games to determine a winner.

$$\text{games} = 1 + m - 2 = m - 1 \checkmark$$

□

Corollary You need $n-1$ comparisons to find the largest (smallest) value in an unsorted array.

Fundamental Theorem of Arithmetic

Every $n > 1$ can be factored as the product of prime numbers.

Pf by induction on n.
(strong)

Proof by contradiction

Suppose that I want to prove some statement p is true.

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1) Assume $\neg p$.

2) Show this leads to a contradiction

3) Conclude p must be true.

$p \vee \neg p$ always true

$\neg p$ is false or leads to a contradiction

$$\frac{\neg p \text{ is false or leads to a contradiction}}{p \text{ must be true}}$$

Example

Claim: $\sqrt{2}$ is irrational

Proof

by contradiction:

Assume $\sqrt{2}$ is rational

So $\sqrt{2} = a/b$

and a and b have no common divisors

$\sqrt{2}\, b = a$

$2b^2 = a^2 \quad (*)$

a^2 is even

$\Rightarrow$ a is even

So $a = 2c$

by $(*)$ $2b^2 = (2c)^2 = 4c^2$

So $b^2 = 2c^2$

b^2 is even $\Rightarrow$ b is even

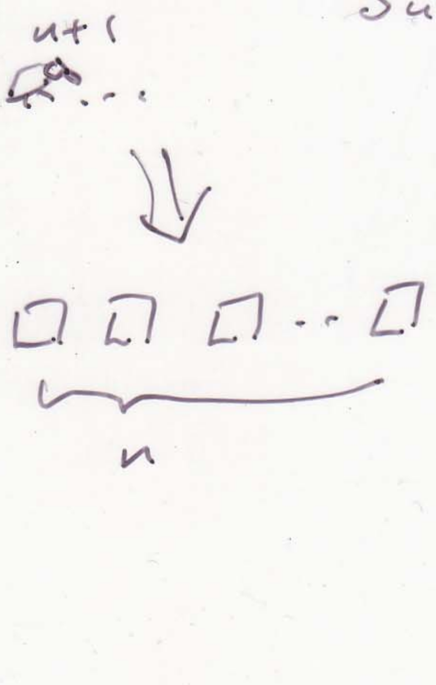
So $b = 2d$

This is a contradiction (-X-)
with a, b sharing no
common divisors

So $\sqrt{2}$ is irrational.

Pigeonhole Principle

Suppose



$n+1$ pigeons
n pigeonholes

$\exists$ a hole with
at least 2 pigeons.

Generalized Pigeohole Principle

Suppose $m \cdot n + 1$ pigeons

n pigeonholes

$\exists$ a pigeonhole with

$\geq m+1$ pigeons.

Proof (by contradiction)

Assume

$\forall$ pigeonholes, pigeon amount $< m+1$

$(\leq m)$

# of pigeons in total

$$= \sum_{\text{holes } h} (\text{\# pigeons in } h)$$

$$\leq \sum_{\text{holes } h} m = m \cdot n$$

$\times$ since there are $mn+1$ pigeons

$\square$