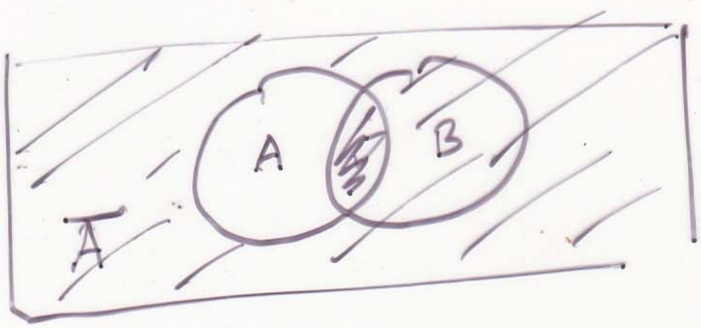
CS 222 Lec 9

Sets ...

operations: $\cap, \cup, \setminus, \overline{}$



$$A \setminus B = A \cap \overline{B}$$

$|A|$ = size of A

* $|A \cup B| = |A| + |B| - |A \cap B|$

The (Cartesian) Product of Sets

Defn $A \times B = \left\{ (a,b) \;\middle|\; \begin{array}{l} a \in A, \\ b \in B \end{array} \right\}$

e.g. $A = \{1, 2\}$

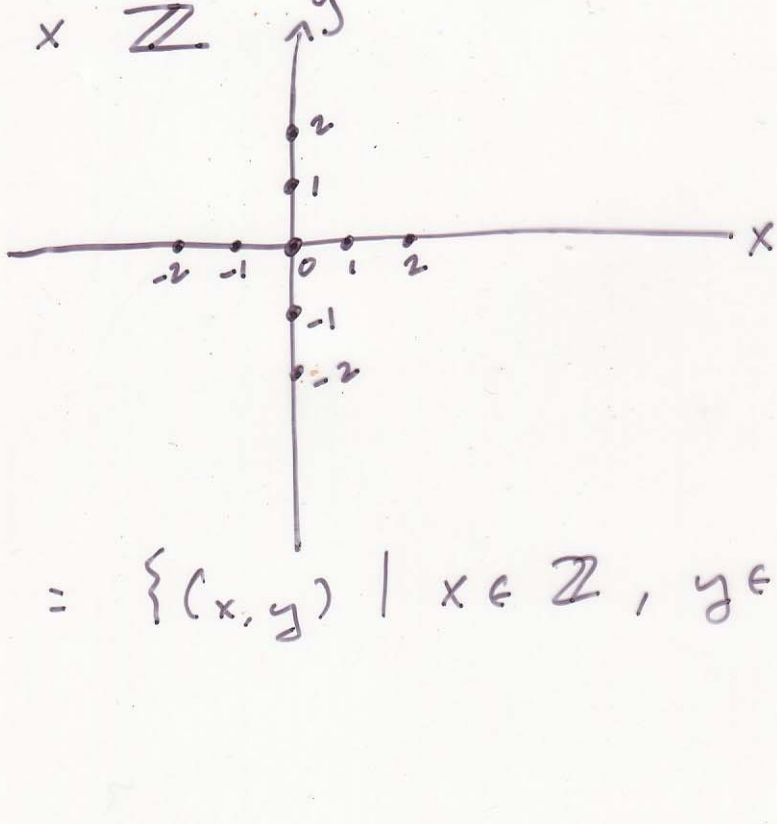
$B = \{a, b, c\}$

$$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$$

A, B finite:

$$|A \times B| = |A| \cdot |B|$$

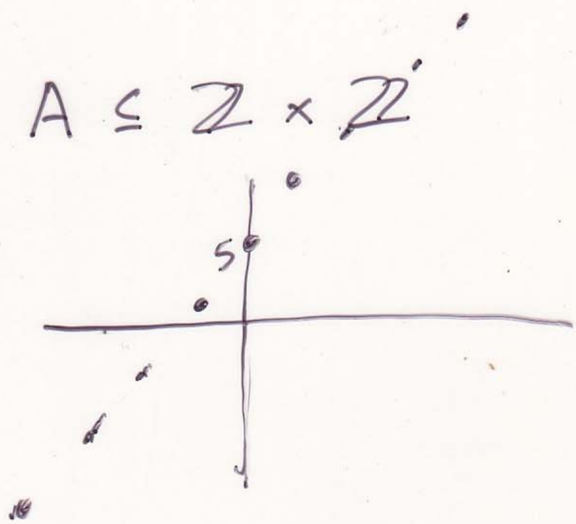
e.g. $\mathbb{Z} \times \mathbb{Z}$



$$= \{(x, y) \mid x \in \mathbb{Z}, y \in \mathbb{Z}\}$$

e.g. $A = \left\{ (x,y) \mid \begin{array}{l} x \in \mathbb{Z},\ y \in \mathbb{Z} \\ y = 2x+5 \end{array} \right\}$

$A \subseteq \mathbb{Z} \times \mathbb{Z}$



Powersets

Let A be a set.

<u>Defn</u> $\mathcal{P}(A) = \{ S \mid S \subseteq A \}$

= set of all subsets of A

$A = \{a, b\}$

$\mathcal{P}(A) = \{ \emptyset, \{a\}, \{b\}, \{a,b\} \}$

Alternative notation:

$$\mathcal{P}(A) = 2^{A}$$

Suppose $|A|$ is finite.

$$|A| = c$$

Then $|\mathcal{P}(A)| = 2^{c}$.

$$A = \{ a_1, a_2 \quad \ldots \quad a_c \}$$

$$\left.\begin{array}{cccccccc}
1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \cdots & & & & & 0 \\
 & & & & & & & \\
1 & 1 & 1 & \cdots & & & & 1
\end{array}\right\}$$

Observe: each binary # leads to a different subset. There are 2^c such binary numbers.

e.g.

$$2^{\mathbb{Z}} = ?$$

$= \mathcal{P}(\mathbb{Z}) =$ set of all subsets of the integers

$$= \{ \{-1, 5, 8\}, \ldots \mathbb{Z} \}$$

<u>infinite sets</u>

cardinality $\approx$ size

$$|\mathbb{Z}|$$

$$|\mathbb{R}|$$

$$|\mathbb{Z}| < |\mathbb{R}| = |2^{\mathbb{Z}}|$$