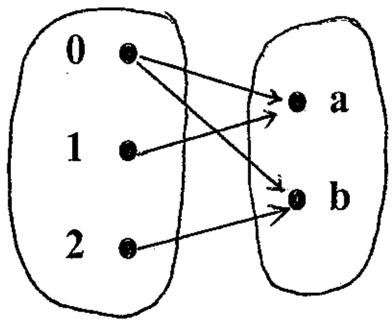
Chapter 8 Relations

8.1, 8.2

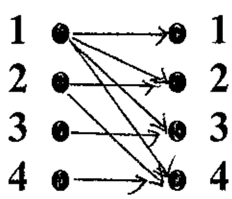
Def. Let A and B be sets. A <u>binary relation</u> from A to B is a subset of A x B.

Ex A B



R	a	b
0	x	x
1	x	
2		x

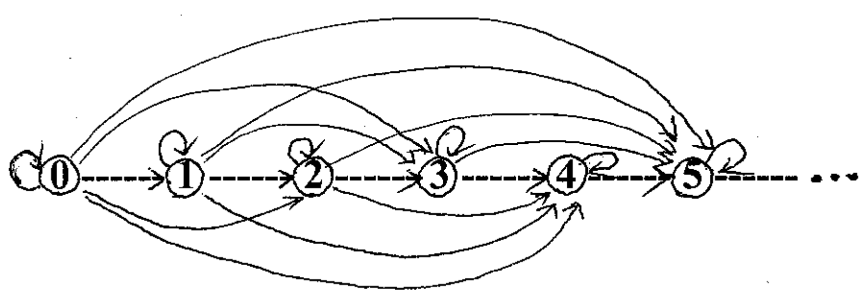
Ex 4. A = {1, 2, 3, 4}
R = {(a,b) | a divides b}



R	1	2	3	4
1	x	x	x	x
2		x		x
3			x	
4				x

- **Reflexive**
  - if for all $a \in A$, $(a,a) \in R$

Ex. $\leq$ on N



- **Symmetric**
  - **(b,a) ∈ R whenever (a,b) ∈ R**

Ex. R = {(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)}

- **Transitive**
  - **(a,b) ∈ R and (b,c) ∈ R, then (a,c) ∈ R**

Ex. R = {(1,1), (2,3), (3,4), (2,4)}
R = {(1,3), (3,2)}

Combining Relations

Ex 17. **A = {1,2,3},** **B = {1,2,3,4}**
R_1= {(1,1),(2,2),(3,3)} **R_2= {(1,1),(1,2),(1,3),(1,4)}**

$R_1 \cup R_2$ =
$R_1 \cap R_2$ =
$R_1 - R_2$ =
$R_2 - R_1$ =

Ex 20. composite
{1,2,3} → {1,2,3,4} **{1,2,3,4} → {0,1,2}**
R={(1,1),(1,4),(2,3),(3,1),(3,4)} **S={(1,0),(2,0),(3,1),(3,2),(4,1)}**

S o R = {(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)}

n-ary Relations = subset of $A_1 \times A_2 \times A_3 \ldots A_n$

Ex. R: N x N x N (degree 3)
(a, b, c) where $a < b < c$.

$(1, 3, 4) \in R$ $(2, 4, 3) \notin R$

Relational Database

Student

Name	ID-number	Major	GPA
Ackermann	231455	CS	3.88
Adams	888323	Physics	3.45
Chou	102147	CS	3.49
Goodfriend	453876	Math	3.45
Rao	678543	Math	3.90
Stevens	786576	Psychology	2.99

Primary key

Operations:

- selection
- projection
- join

SQL

```
SELECT  Name
FROM    Student
WHERE   GPA > 3.5
```

Ans. Ackermann
Rao

8.3 Representation of Relation

Ex. $A = \{a_1, a_2, a_3\}$

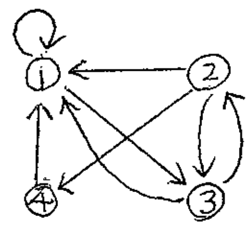
$B = \{b_1, b_2, b_3, b_4, b_5\}$

$R = \{(a_1,b_2),(a_2,b_1),(a_2,b_3),(a_2,b_4),(a_3,b_1),(a_3,b_3),(a_3,b_5)\}$

(1) Matrix

$$M_R = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{ccccc} b_1 & b_2 & b_3 & b_4 & b_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

(2) Digraph



$R = \{(1,1),(1,3),(2,1),(2,3),(2,4),(3,1),(3,2),(4,1)\}$ on $\{1,2,3,4\}$

8.4 Closures of Relations

(1) **Reflexive closure:** add (a,a) if $(a,a) \notin R$

Ex. $R = \{(a,b) \mid a < b\}$

Reflexive closure $= \{(a,b) \mid a \leq b\}$

(2) **Symmetric closure:** $R \cup R^{-1}$ ($R^{-1} = \{(b,a) \mid (a,b) \in R$

Ex. $R = \{(a,b) \mid a > b\}$

$R \cup R^{-1} = \{(a,b) \mid a > b\} \cup \{(b,a) \mid a > b\} = \{(a,b) \mid a \neq b\}$

(3) **Transitive closure = Connectivity relation**

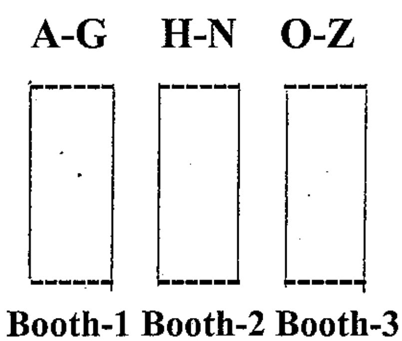
Connectivity relation $R^* = \bigcup_{n=1}^{\infty} R^n$ // there is a path //

Note. Warshall's algorithm

8.5 Equivalence Relation

Def. reflexive, symmetric, transitive

Ex. Registration based on last name



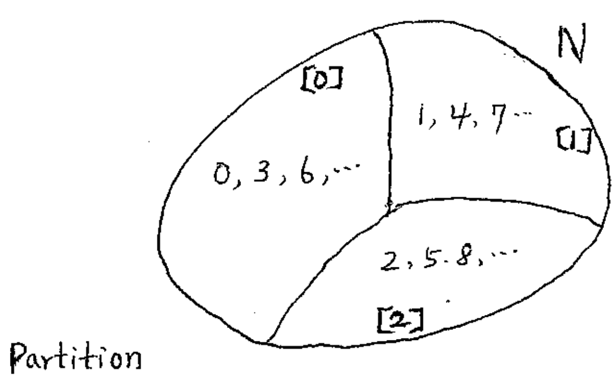
reflexive: xRx
symmetric: xRy, yRx
transitive: xRy, yRz, xRz

Equivalence Class

Def. Let R be an equivalence relation on a set A.
The set of all elements that are related to an element a of A is called the equivalence class of a.

Ex. What are the equivalence class for congruence modulo 3 on N?

$[0]_3 = \{0, 3, 6, 9, \dots\}$
$[1]_3 = \{1, 4, 7, 10, \dots\}$
$[2]_3 = \{2, 5, 8, 11, \dots\}$



Partition

A partition π of a set S is a collection of <u>disjoint</u> <u>nonempty</u> subset of S that have S as their union.

Note.

1. Each element of π is nonempty.
2. Each element appears in <u>exactly one</u> partition.
3. $\cup\pi = A$

Ex. $\mathbb{N} = \{ \mathbb{O}, \mathbb{E} \}$

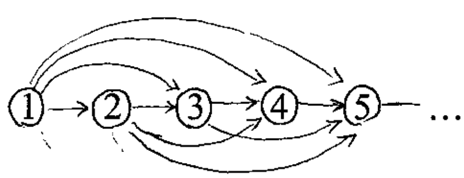
Ex. A = { a, | b, c, | d }
A = { a, b, | c, d }

Ex. 14
π : congruence modular 4 on positive integers

$[0]_4 = \{4, 8, 12, \ldots\}$
$[1]_4 = \{1, 5, 9, \ldots\}$
$[2]_4 = \{2, 6, 10, \ldots\}$
$[3]_4 = \{3, 7, 11, \ldots\}$

Partial Order: reflexive, anti-symmetric, and transitive

Ex. $\leq$ on Z^+



Ex. $<$ on Z^+