Graphs

Chapter 9

Algorithms Using Weighted Graphs

- Finding the shortest path from a vertex to all other vertices
  - Solution formulated by Dijkstra

Dijkstra's Algorithm

```
1.  Initialize S with the start vertex, s, and V–S with the remaining vertices.
2.  for all v in V–S
3.          Set p[v] to s.
4.          if there is an edge (s, v)
5.                  Set d[v] to w(s, v).
            else
6.                  Set d[v] to ∞.
7.  while V–S is not empty
8.          for all u in V–S, find the smallest d[u].
9.          Remove u from V–S and add u to S.
10.         for all v adjacent to u in V–S
11.                 if d[u] + w(u, v) is less than d[v].
12.                         Set d[v] to d[u] + w(u, v).
13.                         Set p[v] to u.
```

Algorithms Using Weighted Graphs (continued)

- A minimum spanning tree is a subset of the edges of a graph such that there is only one edge between each vertex, and all of the vertices are connected
- The cost of a spanning tree is the sum of the weights of the edges
- We want to find the minimum spanning tree or the spanning tree with the smallest cost
- Solution formulated by R.C. Prim and is very similar to Dijkstra's algorithm

Prim's Algorithm

Prim's Algorithm for Finding the Minimum Spanning Tree

```
1.  Initialize S with the start vertex, s, and V–S with the remaining vertices.
2.  for all v in V–S
3.      Set p[v] to s.
4.      if there is an edge (s, v)
5.          Set d[v] to w(s, v).
        else
6.          Set d[v] to ∞.
7.  while V–S is not empty
8.      for all u in V–S, find the smallest d[u].
9.      Remove u from V–S and add it to S.
10.     Insert the edge (u, p[u]) into the spanning tree.
11.     for all v in V–S
12.         if w(u, v) < d[v]
13.             Set d[v] to w(u, v).
14.             Set p[v] to u.
```

Warsall's Algorithm
Finding the reachability matrix in a directed graph.

```
for(int i = 0; i<n; i++)
    for(int j=0; j<n; j++)
        for(int k=0; k<n; k++)
            if(!adj[j][k])
                adj[j][k] = adj[j][i] && adj[i][k];
```

Floyd-Warshall's Algorithm for shortest path

Same as Warshall's reachability matrix solution but the logic in the loop is not using && but finding the minimum

adj[i][k] = mininum(adj[j][k] || (adj[j][i] + adj[i][k]));

Chapter Review

- A graph consists of a set of vertices and a set of edges
- In an undirected graph, if (u,v) is an edge, then there is a path from vertex u to vertex v, and vice versa
- In a directed graph, if (u,v) is an edge, then (v,u) is not necessarily an edge
- If there is an edge from one vertex to another, then the second vertex is adjacent to the first
- A graph is considered connected if there is a patch from each vertex to every other vertex

Chapter Review (continued)

- A tree is a special case of a graph
- Graphs may be represented by an array of adjacency lists
- Graphs may be represented by a two-dimensional square array called an adjacency matrix
- A breadth-first search of a graph finds all vertices reachable from a given vertex via the shortest path
- A depth-first search of a graph starts at a given vertex and then follows a path of unvisited vertices until it reaches a point where there are no unvisited vertices that are reachable

Chapter Review (continued)

- A topological sort determines an order for starting activities which are dependent on the completion of other activities
- Dijkstra's algorithm finds the shortest path from a start vertex to all other vertices
- Prim's algorithm finds the minimum spanning tree for a graph