CHAPTER 0. Foundation

Mathematical Problems

- Numerical Problems
  - $\sqrt{2}$
  - linear system
  - integration
- Non Numerical problems
  - sorting/searching

input data $\xrightarrow[\text{(algorithm)}]{\text{functional connection}}$ output data

Algorithm

complete description of well-defined operations

① definite – clear and unambiguous

② effective – each instruction is sufficiently basic that it can be done with only pencil and paper.

③ finite – terminate.

Program

expression of an algorithm in a programming language.

Algorithm

(1) Expression of algorithm

1. step-by-step in English
2. Flow chart
3. Program listing
4. Algol-like language (pigeon Algol) (Psendo Algol)

(2) Analysis of algorithm

- frequency counting

Example

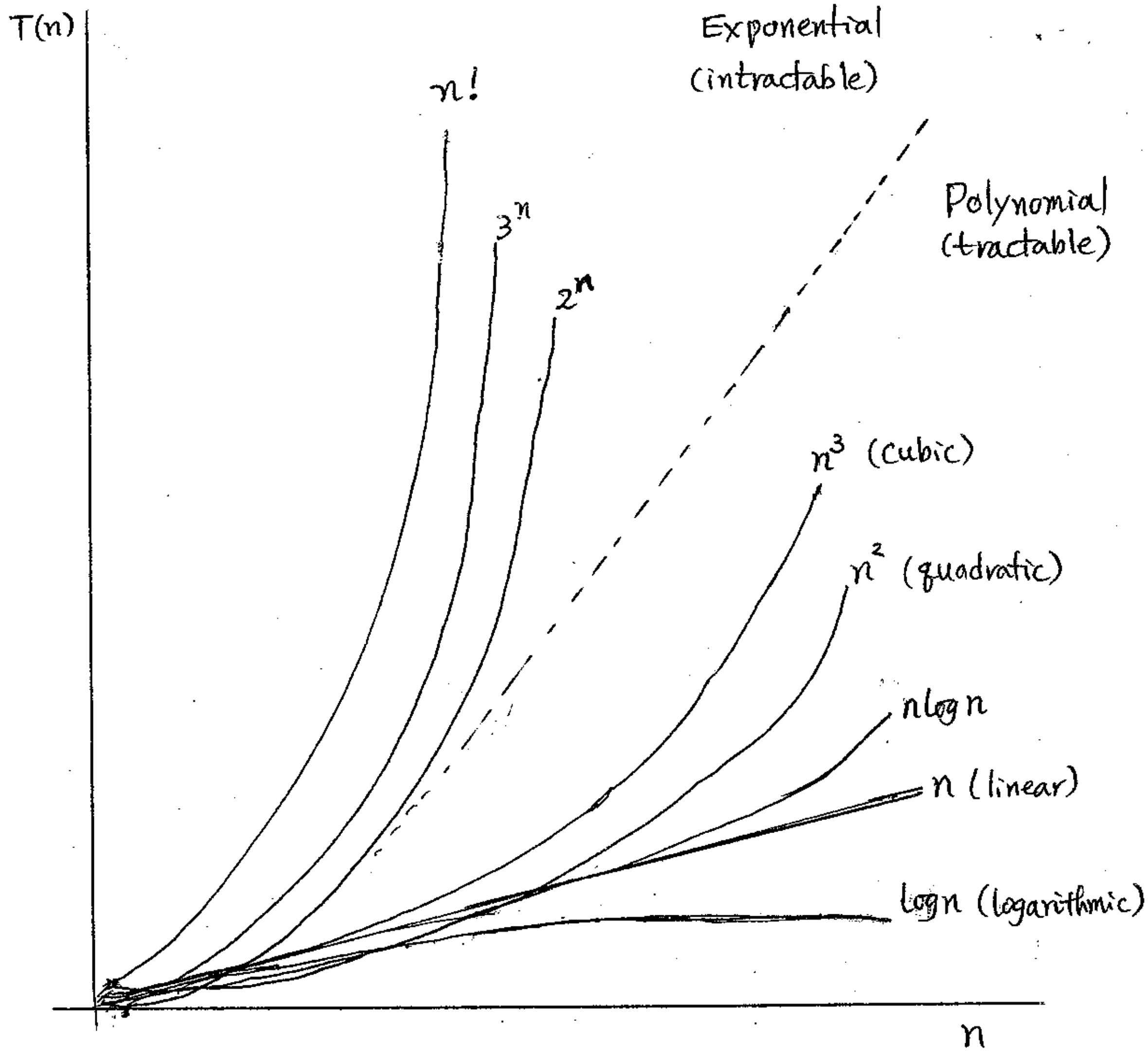
```
sum ← 0
for i ← 1 to n do
    for j ← 1 to i do
        sum ← sum + 1
    endfor
endfor
print, sum    ← ***
```

for $n=10$, sum will be $\boxed{}$.

Time-complexity.

$$T(n) = \frac{1}{2} n(n+1)$$
$$= \frac{1}{2} n^2 + \frac{1}{2} n$$

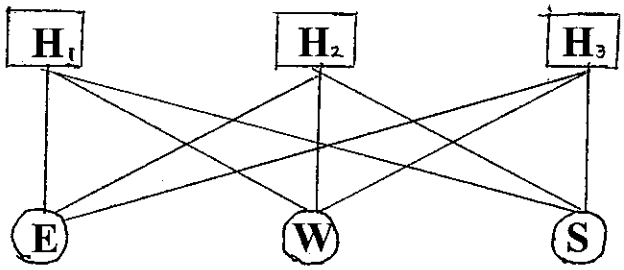
$T(n) = O(n^2)$: rate-of-growth



T(n)
Exponential
(intractable)
n!
3^n
2^n
Polynomial
(tractable)
n^3 (cubic)
n^2 (quadratic)
nlog n
n (linear)
logn (logarithmic)
n

Planarity Test

$|E| \le (3n-6)$

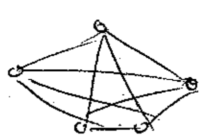


$K_{3,3}$

Algorithm	Complexity	Time for n=100 and k = 10 ms	N 1 min	N 1 hr	
Kuratowski (1930)	$O(n^6)$	325 yr	4	8	
Goldstein (1963)	$O(n^3)$	2.8 hr	18	71	
Lempet et al (1967)	$O(n^2)$	100 sec	77	600	} — adjacency matrix
Hopcroft & Tarjan(1971)	$O(n \log n)$	7 sec	643	24673	} — adjacency list
Tarjan (1971)	$O(n)$	1 sec	6000	360,000	

Note. Planarity 조건 $|E| \le (3n-6)$

Euler's formula (1736) : regions + vertices = edges + 2



K_5

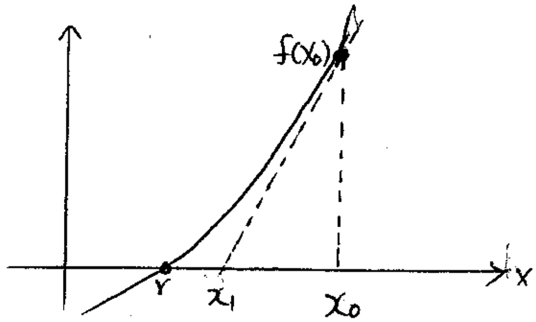
A graph G is planar iff G does not have a subgraph homomorphic to $K_{3,3}$ nor K_5.

Ex. Find $\sqrt{2}$

[1] Hand-calculation

	1. 4 1 4 2
1	2
1	1
24	100
4	96
281	400
1	281
2824	11900
4	11296
28282	60400
2	56564
⋮	⋮

[2] Newton's Method



slope:

$$f'(x_0) = \frac{f(x_0)}{x_0 - x_1}$$

$$X_1 = X_0 - \frac{f(X_0)}{f'(X_0)}$$

$$X_2 = X_1 - \frac{f(X_1)}{f'(X_1)}$$

$$\vdots \quad \vdots \quad \vdots$$

Example. $F(x) = X^3 - X + 1$, $X_0 = 1$

$$X_1 = X_0 - \frac{f(X_0)}{f'(X_0)} \qquad f'(X) = 3X^2 - 1,\ f'(1) = 2,\ f(1) = 1$$

$\therefore\ X_1 = 1 - ½ = ½$

$X_2 = 3$

$\vdots$

Convergence Analysis

$$|X_{n+1} - r| \leq c\,|X_n - r|^2 \qquad \text{quadratic convergence}$$

Note. proof with Taylor series

Application.

$$\sqrt{N} \qquad \sqrt[3]{N}$$

How to find $\sqrt{2}$?

Note. $\sqrt{2}$ is zero of $f(x) = x^2 - 2$

$F(x) = x^2 - 2$

$F'(x) = 2x$

Newton's method:

$$X_{i+1} = X_i - \frac{X_i^2 - 2}{2X_i}$$

$$= X_i - \frac{X_i}{2} + \frac{2}{2X_i}$$

$$= \boxed{\tfrac{1}{2}\left(X_i + \frac{2}{X_i}\right)}$$

Let $X_0 = 2$

$X_1 = \frac{1}{2}\left(2 + \frac{2}{2}\right) = 1.5$

$X_2 = \frac{1}{2}\left(1.5 + \frac{2}{1.5}\right) = \frac{1}{2}(1.5 + 1.3333) = 1.4167$

$X_3 = \frac{1}{2}\left(1.4167 + \frac{2}{1.4167}\right) = 1.4142$

2.000000000000000
1.500000000000000
1.416666666666667
1.414215686274510
1.414213562374690
1.414213562373095

$X_0 = 1000$

1000.000000000000000
500.001000000000000
250.002499996000010
125.005249958000470
62.510624643017032
31.271309602062194
15.667632994868366
7.897642347856358
4.075441240519499
2.283092824392554
1.579548752406015
1.422866579578668
1.414239873591531
1.414213562617849
1.414213562373095

TABLE I. Convergence to Sqrt(2)